CHAPTER 9

Chapter Overview

Quadratic and Exponential Functions

Standards-Based Lesson Plan

LESSONS AND OBJECTIVES	California Standards	Pacing Your Lessons: 40-50 Minute Periods	Pacing Your Lessons: 90-Minute Periods
Explore 9-1 **Graphing Calculator Lab: Exploring Graphs of Quadratic Functions** (p. 470) **9-1 Graphing Quadratic Functions** (pp. 471–477) • Graph quadratic functions. • Find the equation of the axis of symmetry and the coordinates of the vertex of a parabola. **Extend 9-1** **Graphing Calculator Lab: The Family of Quadratic Functions** (pp. 478–479)	1A21.0	3	2
9-2 Solving Quadratic Equations by Graphing (pp. 480–485) • Solve quadratic equations by graphing. • Estimate solutions of quadratic equations by graphing.	1A21.0 1A22.0	2	1
9-3 Solving Quadratic Equations by Completing the Square (pp. 486–491) • Solve quadratic equations by finding the square root. • Solve quadratic equations by completing the square.	1A14.0	2	1
9-4 Solving Quadratic Equations by Using the Quadratic Formula (pp. 493–499) • Solve quadratic equations by using the Quadratic Formula. • Use the discriminant to determine the number of solutions for a quadratic equation. **Extend 9-4** **Algebra Lab: Applying Quadratic Equations** (pp. 500–501)	1A19.0 1A20.0 1A22.0	2	1
9-5 Exponential Functions (pp. 502–508) • Graph exponential functions. • Identify data that displays exponential behavior.	2A12.0	2	1
Explore 9-6 **Algebra Lab: Investigating Exponential Functions** (p. 509) • Use paper stacking to investigate an exponential function. **9-6 Growth and Decay** (pp. 510–514) • Solve problems involving exponential growth. • Solve problems involving exponential decay. **Extend 9-6** **Graphing Calculator Lab: Curve Fitting** (pp. 515–516)	2A12.0	3	1.5
REVIEW		1	0.5
ASSESSMENT		1	0.5
	TOTAL	**16**	**8.5**

*The complete **Assessment Planner** for Chapter 9 is provided on page 469.*

** Begin Chapter 10 in the second half of the period.*

Professional Development

California Standards Vertical Alignment

Before Chapter 9

Related Topics from Grade 7

- Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems Standard 7AF3.1
- Use order of operations to evaluate algebraic expressions Standard 7AF1.2

Chapter 9

Topics from Algebra I

- Solve a quadratic equation by factoring or completing the square
- Graph quadratic functions and know that their roots are the x-intercepts
- Determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points
- Apply quadratic equations to physical problems

See individual lessons for the specific Standards covered.

After Chapter 9

Preparation for Algebra II

- Solve and graph quadratic equations and quadratic equations in the complex number system Standard 2A8.0
- Use the definition of logarithms to translate between logarithms in any base Standard 2A13.0

Back-Mapping

California Algebra 1 was conceived and developed with the final result in mind, student success in Algebra I and beyond. The authors, using the California Mathematics Standards as their guide, developed this brand-new series by "back-mapping" from the desired result of student success in Algebra I and beyond. McGraw-Hill's *California Geometry, California Algebra 2,* and *California Algebra Readiness* were developed utilizing the same philosophy.

What the Research Says…

According to Ellington (2003), students' operational and problem-solving skills improve when calculators are an integral part of testing and instruction.

- In Lessons 9-1A and 9-1B, graphing calculators are used to explore properties of quadratic functions.
- In Lesson 9-6B, graphing calculators are used in data analysis to do curve fitting.
- Graphing calculators are used for problem-solving in Lessons 9-2, 9-4, and 9-5.

[Source: Ellington, A.J. (2003). "A Meta-Analysis of the Effects of calculators on Students' Achievement and Attitude Levels in Precollege Mathematics Classes," *Journal for Research in Mathematics Education*, 34(5), pp. 433–463.]

McGraw Hill Professional Development

Targeted professional development has been articulated throughout the ***California Mathematics: Concepts, Skills, and Problem Solving*** series. The **McGraw-Hill Professional Development Video Library** provides short videos that support the Key Standards. For more information, visit ca.algebra1.com.

Model Lessons

Instructional Strategies

CHAPTER 9

Technology Solutions

Teacher Resources

TeacherWorks™ All-in-One Planner and Resource Center

All of the print materials from the Classroom Resource Masters are available on your TeacherWorks™ CD-ROM.

BL = Below Grade Level OL = On Grade Level AL = Above Grade Level ELL = English Language Learner

				Chapter Resource Masters	9-1	9-2	9-3	9-4	9-5	9-6
BL	OL		ELL	Lesson Reading Guide	5	12	19	26	33	41
BL	OL		ELL	Study Guide and Intervention*	6	13	20	27	34	42
BL	OL			Skills Practice*	8	15	22	29	36	44
	OL	AL		Practice*	9	16	23	30	37	45
	OL	AL		Word Problem Practice*	10	17	24	31	38	46
	OL	AL		Enrichment	11	18	25	32	39	47
	OL	AL		Calculator and Spreadsheet Activities					40	
	OL	AL		Chapter Assessments*	49–70					
BL	OL	AL		5-Minute Check Transparencies	✓	✓	✓	✓	✓	✓
BL	OL			Teaching Algebra with Manipulatives	✓		✓		✓	✓

**Also available in Spanish.*

AssignmentWorks

Differentiated Assignments, Answers, and Solutions

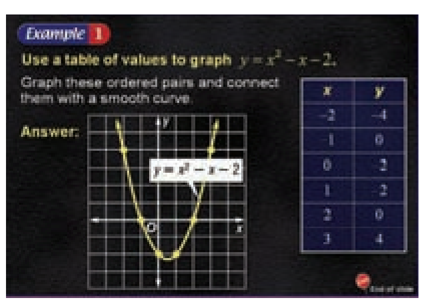
- Print a customized assignment worksheet using the Student Edition exercises along with an answer key or worked-out solutions.
- Use default lesson assignments as outlined in the Differentiated Homework Options in the Teacher Wraparound Edition.
- Includes modified questions from the Student Edition.

Interactive Classroom

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- In-Class Examples
- Your Turn Exercises*
- 5-Minute Check Transparencies*
- Links to Online Study Tools
- Concepts in Motion

**compatible with response pad technology*



ExamView® Assessment Suite

ExamView® Assessment Suite lets you

- Create, edit, and customize tests and worksheets using QuickTest Wizard
- Create multiple versions of tests and modify them for a desired level of difficulty
- Translate from English to Spanish and vice versa
- Build tests aligned with your state standards
- Track students' progress using the Teacher Management System

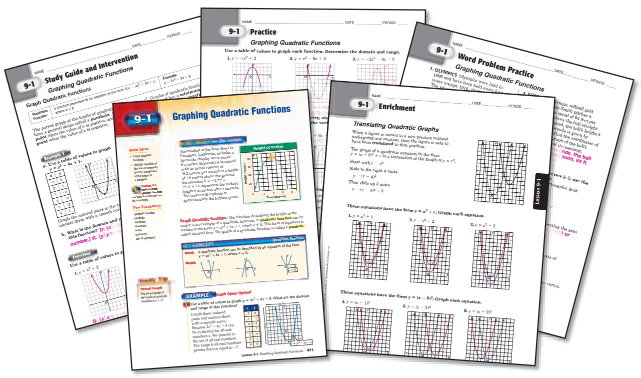
Student Resources

StudentWorks™ Plus

Textbook, Audio, Workbooks, and more

This CD-ROM is a valuable resource for students to access content online and use online resources to continue learning Chapter 9 concepts. Includes:

- Complete Student Editions in both English and Spanish
- English audio integrated throughout the text
- Links to Concepts in Motion, Personal Tutor, and other online resources
- Access to all student worksheets
- Daily Assignments and Grade Log



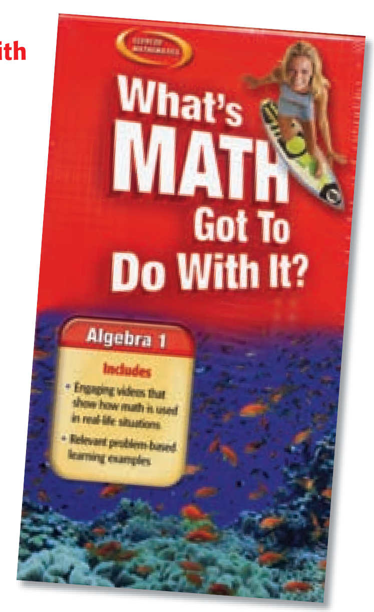
Super DVD

The Super DVD contains two Glencoe multimedia products.

MindJogger Plus An alternative review of concepts in which students work as teams in a game show format to gain points for correct answers.

What's Math Got to Do With It? Real Life Math Videos
Engaging video that shows students how math is used in everyday situations

Unit 3 theme: Polynomials and Nonlinear Functions



Internet Resources

Math Online ca.algebra1.com

TEACHER	PARENT	STUDENT	Online Study Tools
	●	●	Online Student Edition
●	●	●	Multilingual Glossary
			Lesson Resources
	●	●	BrainPOP®
●	●	●	Concepts in Motion
●	●	●	Extra Examples
	●	●	Other Calculator Keystrokes
●			Problem of the Week Cards
	●	●	Real-World Careers
	●	●	Self-Check Quizzes
			Chapter Resources
	●	●	Chapter Readiness
	●	●	Chapter Test
	●	●	Standardized Test Practice
	●	●	Vocabulary Review/Chapter Review Activities
			Unit Resources
●		●	Cross-Curricular Internet Project
			Other Resources
●			Dinah Zike's Foldables
	●	●	Hotmath Homework Help
●			Key Concepts
●	●	●	Meet the Authors
	●	●	Personal Tutor
●			Project CRISS℠
	●	●	Scavenger Hunts and Answer Sheets
●			Vocabulary PuzzleMakers

Focus on Mathematical Content

Big Idea for Chapter 9: *Quadratic and Exponential Functions*

In some situations, quadratic functions can be used to model non-linear problems. Depending on the situation, quadratic equations may be solved in different ways. For example, a problem involving physical motion may be solved by graphing, allowing the situation to be visualized. However, when solutions cannot be determined from a graph, other methods, such as completing the square and using the Quadratic Formula, may be appropriate. There are also situations that may be modeled by functions that are neither linear nor quadratic. Exponential functions may be used to describe changes in population growth, to solve compound interest problems, and to model other situations that display exponential behavior.

Why It's Important

For This Chapter

The lessons in this chapter introduce students to quadratic and exponential functions.

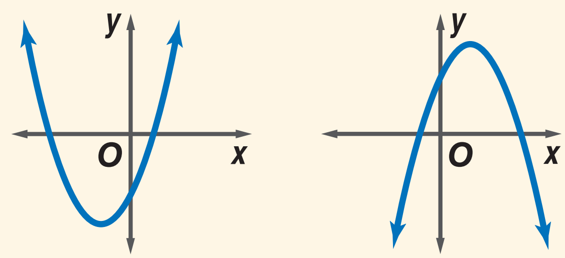
- What is a parabola? (Lesson 9-1)
- How many roots does a quadratic equation have and how do you find them? (Lesson 9-2)
- How do you complete the square for a quadratic expression of the form $x^2 + bx$? (Lesson 9-3)
- According to the Quadratic Formula, what are the solutions of a quadratic equation in the form $ax^2 + bx + c = 0$? (Lesson 9-4)
- How can you determine whether a set of data displays exponential behavior? (Lesson 9-5)
- What is the difference between exponential growth and exponential decay? (Lesson 9-6)

After This Chapter

- In future studies of math, students can use the Quadratic Formula to solve any second-degree polynomial equation.
- In many fields, including science and finance, exponential functions can be applied to solve problems involving growth and decay.

9-1 Graphing Quadratic Functions

The standard form of a quadratic function is $y = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is a symmetrical curve called a *parabola*. If a is positive, the parabola opens upward and the vertex is the minimum of the function. If a is negative, the parabola opens downward and the vertex is the maximum of the function.



If the graph of a parabola is folded in half, the two sides match exactly. The line that divides a parabola in half is called the *axis of symmetry,* and the *vertex* is the only point on the parabola that is on the axis of symmetry. The equation for the axis of symmetry for the graph of $y = ax^2 + bx + c$ is $x = -\frac{b}{2a}$. You can find the x-coordinate of the vertex by knowing the axis of symmetry. However, to find the y-coordinate, you must substitute the value of x into the quadratic equation.

9-2 Solving Quadratic Equations by Graphing

The solutions of a quadratic equation are called *roots.* The roots can be found by graphing the related quadratic function and finding the x-intercepts of the parabola. Quadratic equations always have two roots. The roots can be two real roots, a double real root, or two roots that are not real numbers.

A quadratic equation has:

- two real roots when the parabola crosses the x-axis at two distinct points,
- a double real root when the vertex of the parabola is on the x-axis, or
- no real roots when the parabola does not intersect the x-axis.
- Sometimes the roots are not integers and must be estimated from the graph.

9-3 Solving Quadratic Equations by Completing the Square

If the quadratic expression on one side of the equation in the form $ax^2 + bx + c = n$ is a perfect square, the equation can be solved by taking the square root of each side. However, since few quadratic expressions are perfect squares, a method called *completing the square* may be used.

To solve an equation of the form $x^2 + bx + c = 0$ by completing the square, isolate the x^2 and bx terms on one side of the equation. Find half of b and square it. Then add this amount to each side of the equation. Solve by factoring and taking the square root of each side. If the coefficient of x^2 is not 1, divide each term by the coefficient before completing the square.

9-4 Solving Quadratic Equations by Graphing

A quadratic equation in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$, can be solved using the Quadratic Formula. The coefficients a and b and the constant c are substituted into the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Then the expression is simplified to determine the solutions.

The expression under the radical sign, $b^2 - 4ac$, is called the *discriminant.* The value of the discriminant can be used to determine the number of real roots for a quadratic equation.

- A positive discriminant indicates two real roots.
- A negative discriminant indicates that there are no real roots.
- If the discriminant is 0, there is one real root.

9-5 Exponential Functions

Exponential functions are neither linear nor quadratic. An exponential function has a variable as an exponent and can be described by an equation of the form $y = ax$, where $a > 0$ and $a \neq 1$. Graphs of exponential functions have distinctive shapes.

- When a is greater than 1, y-values change little for small values of x, but increase quickly as the values of x become greater. The graph rises faster and faster as you move from left to right.
- When $0 < a < 1$, y values decrease as x increases. The graph falls more slowly as the x values increase.
- Two ways to identify exponential functions are to look at the graph and to look for a pattern in the data. If domain values at regular intervals have corresponding range values that have a common factor, the behavior is exponential.

9-6 Growth and Decay

Many real-world problems can be solved using methods for modeling exponential growth and decay. Compound interest is an application of exponential growth. Depreciation is an application of exponential decay.

- Exponential growth can be modeled using the general equation $y = C(1 + r)^t$. The initial amount C increases by the same percent over a given period of time. In the general equation, y represents the final amount, C represents the initial amount, r represents the rate of change expressed as a decimal, and t represents time.
- Exponential decay is a variation of exponential growth. Instead of increasing, the original amount decreases by the same percent over a given period of time. The general equation for exponential decay is $y = C(1 - r)^t$.

CHAPTER 9

Differentiated Instruction

Options for Chapter 9 Lessons

ELL = English Language Learner   AL = Above Grade Level   SS = Struggling Students   SN = Special Needs

Using Interpersonal Skills AL

Use with Lesson 9-1

Place students in small groups. Since there are several tasks involved in graphing quadratic functions, have the group members decide which of the tasks they should complete in order to graph a given function. For example, one member can be responsible for finding the equation for the axis of symmetry, another can substitute values in order to determine points on the graph, and a third member can graph the points and draw the curve of the parabola.

Using Verbal Skills SS SN

Use with Lesson 9-4

Have students come up with a riddle, poem, rap, or other mnemonic device to help them remember how to use the Quadratic Formula, and how to interpret the discriminant. Ask students to share their mnemonic devices with the class.

Connecting Mathematics to Music ELL

Use with Lesson 9-5

Ask students who are familiar with music to examine the different types of notes (whole, half, quarter, etc.) to determine whether the length of the notes exhibits exponential behavior. Have them create posters displaying the different notes, their length, and an exponential expression for determining each note's length.

Noteables™ Interactive Study Notebook with Foldables™

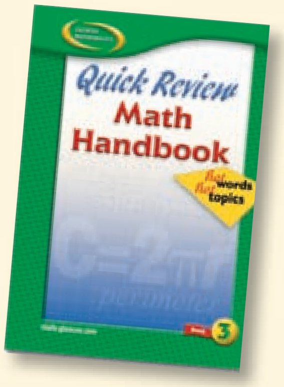
Noteables™ Interactive Study Notebook with Foldables™ is a study organizer that provides helpful steps for students to follow to organize their notes for Chapter 9.

- Students use Noteables to record notes and to complete their Foldables as you present the material for each lesson.
- Noteables correspond to the Examples in the *Teacher Wraparound Edition* and *Interactive Classroom CD-ROM.*

Intervention

Quick Review Math Handbook*

is Glencoe's mathematical handbook for students and parents.



Hot Words includes a glossary of terms.

Hot Topics consists of two parts:

- explanations of key mathematical concepts
- exercises to check students' understanding.

Lesson	Hot Topics Section	Lesson	Hot Topics Section
9-1	6.8	9-4	6.3
9-3	6.3	9-5	6.3

Also available in Spanish

Teacher to Teacher

Linda Hayek
Ralston Public Schools
Omaha, NE

USE BEFORE LESSON 9-4

“I like for my students to discuss all the different ways that a quadratic equation can be solved before presenting the Concept Summary on p. 489. We conduct a class discussion, listing the possibilities when each type might be used. Then we write a summary on our Foldables.”

Reading and Writing in Mathematics

Project CRISS[SM] STUDY SKILL

Power notes can help students organize and outline a lesson or chapter. Students often benefit from making power notes as a cooperative activity. In the outline at the right, Power 1 is the main idea, Power 2 provides details about the main idea, Power 3 provides details about Power 2, and so on.

You may have students copy the power notes at the right and complete them using the information in Chapter 9. Point out that more than one detail can be placed under each power.

1. Solving Quadratic Equations
    2. Graph the Equation
        3. zero roots; if it doesn't intersect the x-axis
        3. one root; if it touches the x-axis at one point
        3. two roots; if it intersects the x-axis at two points
    2. Complete the Square
        3. Find $\frac{1}{2}$ of the coefficient of x.
        3. Square the result from above.
        3. Add the new result to each side of the original equation.
        3. Solve
    2. Use the Quadratic Formula
        3. Rewrite the equation in standard form.
        3. Substitute values into the formula.
        3. Simplify.

CReating **I**ndependence through **S**tudent-owned **S**trategies

CHAPTER 9 Notes

FOLDABLES Study Organizer **Dinah Zike's Foldables™**

Focus As students work through the lessons in this chapter, they write notes and show examples about quadratic and exponential functions.

Teach Have students make and label their Foldables as illustrated.

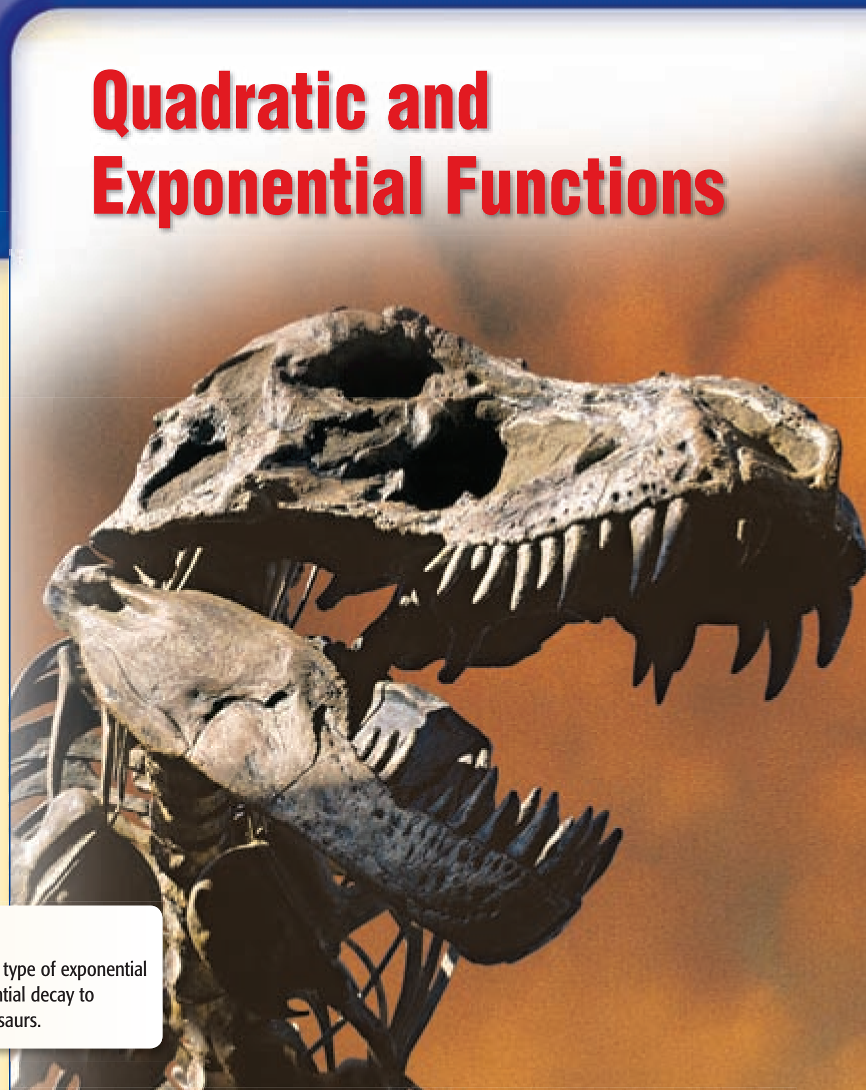
Suggest that students use their Foldables to take notes, define terms, record concepts, and write examples. Ask students to note the order in which the concepts are presented in this chapter. Have them write about why the concepts were presented in that sequence. If they have difficulty recognizing the logic in this sequence, have them outline the key concepts in their own order and justify their reasoning in writing.

When to Use It As students read and study the chapter, have them fill their journals with notes, graphs, and examples of quadratic and exponential functions. *A version of a completed Foldable is shown on p. 517.*

Materials Needed for Chapter 9

- data collection device (Explore 9-1)
- motion sensor (Explore 9-1)
- graphing calculator (Extend 9-1, Lessons 9-1, 9-2, 9-4, 9-5, Extend 9-6)
- algebra tiles (Lesson 9-3)
- scissors (Explore 9-6)
- Internet (Lesson 9-6)

CHAPTER 9 Quadratic and Exponential Functions



BIG Ideas

- **Standard 14.0** Students solve a quadratic equation by factoring or completing the square. (Key)
- **Standard 21.0** Students graph quadratic functions and know that their roots are the *x*-intercepts.
- **Standard 23.0** Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity. (Key)

Key Vocabulary

completing the square (p. 487)
exponential function (p. 502)
parabola (p. 471)
Quadratic Formula (p. 493)

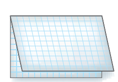
Real-World Link

Dinosaurs Exponential decay is one type of exponential function. Carbon dating uses exponential decay to determine the age of fossils and dinosaurs.

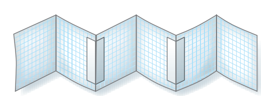
FOLDABLES Study Organizer

Quadratic and Exponential Functions Make this Foldable to help you organize your notes. Begin with three sheets of grid paper.

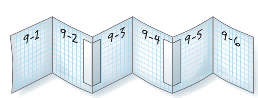
1. **Fold** each sheet in half along the width.



2. **Unfold** each sheet and tape to form one long piece.



3. **Label** each page with the lesson number as shown. Refold to form a booklet.



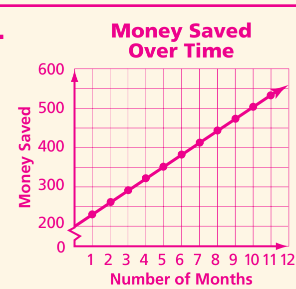
Differentiated Instruction

CRM Student-Built Glossary, pp. 1–2

Students should complete the chart by providing a definition and an example of each term as they progress through Chapter 9. This study tool can also be used to review for the chapter test.

Additional Answer

7.



GET READY for Chapter 9

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at ca.algebra1.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Use a table of values to graph each equation. (Lesson 3-3) (Used in Lesson 9-1.)

1. $y = x + 5$
2. $y = 2x - 3$
3. $y = 0.5x + 1$
4. $y = -3x - 2$
5. $2x - 3y = 12$
6. $5y = 10 + 2x$

1–6. Ch. 9 Answer Appendix.

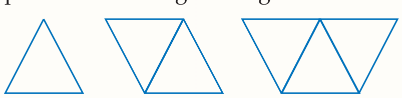
7. **SAVINGS** Suppose you have already saved \$200 toward the cost of a car. You plan to save \$35 each month for the next several months. Graph the equation for the total amount T you will have in m months. $T = 200 + 35m$; see margin for graph.

Determine whether each trinomial is a perfect square trinomial. If so factor it. (Lesson 8-6) (Used in Lesson 9-3.)

8. $t^2 + 12t + 36$ yes; $(t + 6)^2$
9. $a^2 - 14a + 49$ yes; $(a - 7)^2$
10. $m^2 - 18m + 81$ $(m - 9)^2$
11. $y^2 + 8y + 12$ no
12. $9b^2 - 6b + 1$ yes; $(3b - 1)^2$
13. $6x^2 + 4x + 1$ no
14. $4p^2 + 12p + 9$ yes; $(2p + 3)^2$
15. $16s^2 - 24s + 9$ yes; $(4s - 3)^2$

Find the next three terms of each arithmetic sequence. (Lesson 3-4) (Used in Lessons 9-5 and 9-6.)

16. 5, 9, 13, 17, … 21, 25, 29
17. 12, 5, −2, −9, … −16, −23, −30
18. −4, −1, 2, 5, … 8, 11, 14
19. 24, 32, 40, 48, … 56, 64, 72
20. **GEOMETRY** Write a formula that can be used to find the number of sides of a pattern containing n triangles.



3 sides 5 sides 7 sides

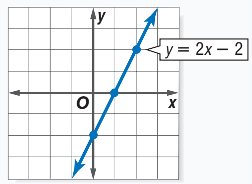
$3 + 2(n - 1)$ or $2n + 1$

QUICKReview

EXAMPLE 1

Use a table of values to graph $y = 2x - 2$.

x	$y = 2x - 2$	y
−1	$2(-1) - 2$	−4
0	$2(0) - 2$	−2
1	$2(1) - 2$	0
2	$2(2) - 2$	2



EXAMPLE 2

Determine whether $x^2 - 22x + 121$ is a perfect square trinomial. If so, factor it.

1. Is the first term a perfect square? yes
2. Is the last term a perfect square? yes
3. Is the middle term equal to $2(1x)(11)$? yes

$x^2 - 22x + 121$ is a perfect square trinomial.

$x^2 - 22x + 121 = (x - 11)^2$

EXAMPLE 3

Find the next three terms of the arithmetic sequence −104, −4, 96, 196, … .

Find the common difference by subtracting successive terms.

$-4 - (-104) = 100$

The common difference is 100.

Add to find the next three terms.

$196 + 100 = 296$, $296 + 100 = 396$, $396 + 100 = 496$

The next three terms are 296, 396, 496.

Diagnostic Assessment

Exercises	California Standards	Intervention
1–7	1A6.0, 1A7.0	SE Review Lesson 3-3, pp. 155–161
8–15	1A11.0, 1A14.0	SE Review Lesson 8-6, pp. 454–460
16–20	1A1.0	SE Review Lesson 3-4, pp. 165–170

ASSESSMENT PLANNER

CHAPTER 9

Formative Assessment

CRM Anticipation Guide, p. 3

Spotting Preconceived Ideas
Students complete this survey to determine prior knowledge about ideas from Chapter 9. Revisit this worksheet after completing the chapter. Also see p. 518.

TWE Lesson Activities

- Ticket Out the Door, pp. 477, 499, 514
- Crystal Ball, p. 508
- Name the Math, p. 491
- Yesterday's News, p. 485

Chapter Checkpoints

SE Mid-Chapter Quiz, p. 492

SE Study Guide and Review, pp. 517–520

SE California Standards Practice, pp. 522–523

CRM Quizzes, pp. 51 and 52

CRM Standardized Test Practice, pp. 68–70

Math Online ca.algebra1.com

- Self-Check Quizzes
- Practice Test
- Standardized Test Practice

Summative Assessment

SE Chapter Practice Test, p. 521

CRM Mid-Chapter Test, p. 53

CRM Vocabulary Test, p. 54

CRM Extended-Response Test, p. 67

CRM Leveled Chapter Tests, pp. 55–66

ExamView® Assessment Suite

KEY

CRM *Chapter 9 Resource Masters*

SE Student Edition

TWE Teacher Wraparound Edition

CD-ROM

EXPLORE 9-1

Lesson Notes

1 Focus

Objective Use a data collection device to conduct an experiment and investigate quadratic functions.

Materials
- graphing calculator
- data collection device and compatible motion detector
- ball

Teaching Tip During this activity, one person will operate the data collection device and the other will toss the ball. To keep the data as accurate as possible, it is important that students synchronize their activities.

2 Teach

Working in Cooperative Groups
Put students in groups of 2 or 3, mixing abilities. Have groups complete the Activity and Analyze the Results 1–4.

- Make sure students know how to use the TRACE feature of the calculator for Analyze the Results 4.
- Students should recognize that the graphs of these experiments are not linear and should look like graphs of quadratic functions.

Practice Have students complete Exercises 5 and 6.

3 Assess

Formative Assessment
Use Exercise 2 to assess whether students can explain why their graphs are not linear.

From Concrete to Abstract
Ask: What do all the graphs in this lab have in common? Sample answer: They all have a U shape.

EXPLORE 9-1

Graphing Calculator Lab

Exploring Graphs of Quadratic Functions

Preparation for Standard 23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity. (Key)

Not all functions are linear. The graphs of nonlinear functions have different shapes. One type of nonlinear function is a *quadratic function*. The graph of a quadratic function is a *parabola*. You use a data collection device to conduct an experiment and investigate quadratic functions.

SET UP the Lab

- Set up the data collection device to collect data every 0.2 second for 4 seconds.
- Connect the motion sensor to your data collection device. Position the motion detector on the floor pointed upward.

ACTIVITY

Step 1 Have one group member hold a ball about 3 feet above the motion detector. Another group member will operate the data collection device.

Step 2 When the person operating the data collection device says "go," he or she should press the start button to begin data collection. At the same time, the ball should be tossed straight upward.

Step 3 Try to catch the ball at about the same height at which it was tossed. Stop collecting data when the ball is caught.

ANALYZE THE RESULTS 1, 4–6. See students' work.

1. The domain contains values represented by the independent variable, time. The range contains values represented by the dependent variable, distance. Use the graphing calculator to graph the data.
2. Write a sentence that describes the shape of the graph. Is the graph linear? Explain.
3. Describe the position of the point on the graph that represents the starting position of the ball. The starting position is the *y*-intercept.
4. Use the TRACE feature of the calculator to find the maximum height of the ball. At what time was the maximum height achieved?
5. Repeat the experiment and toss the ball higher. Compare and contrast the new graph and the first graph.
6. Conduct an experiment in which the motion detector is held at a height of 4 feet and pointed downward at a dropped ball. How does the graph for this experiment compare to the other graphs?

2. Sample answer: It's a parabola. The graph is not linear because it is not a line.

Extending the Concept Introduce the quadratic function $y + x^2$. Show students how to enter it in the Y= LIST. Encourage them to investigate the graph using the TRACE function of their calculators. Have them compare the graph to the graphs of their data in the lab.

9-1 Graphing Quadratic Functions

Main Ideas

- Graph quadratic functions.
- Find the equation of the axis of symmetry and the coordinates of the vertex of a parabola.

Standard 21.0 **Students graph quadratic functions** and know that their roots are the x-intercepts.

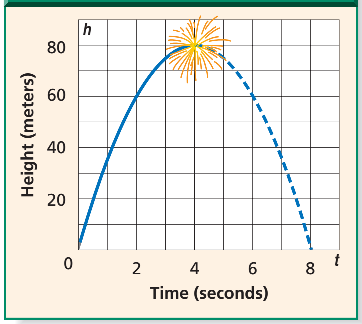
New Vocabulary

quadratic function
parabola
minimum
maximum
vertex
symmetry
axis of symmetry

GET READY for the Lesson

Americafest at the Rose Bowl in Pasadena, California, includes a fireworks display set to music. If a rocket (firework) is launched with an initial velocity of 39.2 meters per second at a height of 1.6 meters above the ground, the equation $h = -4.9t^2 + 39.2t + 1.6$ represents the rocket's height h in meters after t seconds. The rocket will explode at approximately the highest point.

Height of Rocket

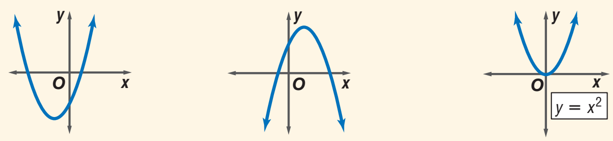


Graph Quadratic Functions The function describing the height of the rocket is an example of a quadratic function. A **quadratic function** can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. This form of equation is called *standard form*. The graph of a quadratic function is called a **parabola**.

KEY CONCEPT — *Quadratic Function*

Words A quadratic function can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.

Models



Study Tip

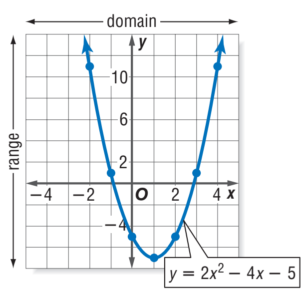
Parent Graph
The parent graph of the family of quadratic functions is $y = x^2$.

EXAMPLE — Graph Opens Upward

1 **Use a table of values to graph $y = 2x^2 - 4x - 5$. What are the domain and range of this function?**

Graph these ordered pairs and connect them with a smooth curve. Because $2x^2 - 4x - 5$ can be evaluated for all real numbers x, the domain is the set of all real numbers. The range is all real numbers greater than or equal to -7.

x	y
-2	11
-1	1
0	-5
1	-7
2	-5
3	1
4	11



9-1 Lesson Notes

1 Focus

Standards Alignment

Before Lesson 9-1
Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems from Standard 7AF3.1

Lesson 9-1
Graph quadratic functions and know that their roots are the x- intercepts from Standard 21.0

After Lesson 9-1
Solve and graph quadratic equations and quadratic equations in the complex number system from Standard 2A8.0

2 Teach

Scaffolding Questions

Have students read *Get Ready for the Lesson.*

Ask:

- Which is more important to the music planners: the height of the firework when it explodes, or the time at which it explodes? The time is more important if the planners want to coordinate the explosions with the music.
- Why must the planners still know the height? The firework explodes at approximately its highest point. When the highest point is found, the value of t at this point is the time at which the firework will explode.

Lesson 9-1 Resources

Chapter 9 Resource Masters

Lesson Reading Guide, p. 5 BL OL ELL
Study Guide and Intervention, pp. 6–7 BL OL ELL
Skills Practice, p. 8 BL OL
Practice, p. 9 OL AL
Word Problem Practice, p. 10 OL AL
Enrichment, p. 11 OL AL

Transparencies

5-Minute Check Transparency 9-1

Additional Print Resources

Noteables™ Interactive Study Notebook with Foldables™
Teaching Algebra with Manipulatives
Science and Mathematics Lab Manual, pp. 61–64

Technology

ca.algebra1.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

Graph Quadratic Equations

Example 1 shows how to graph a quadratic function when the parabola opens upward. **Example 2** shows how to graph a quadratic function when the parabola opens downward.

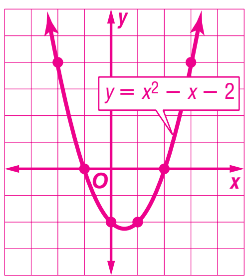
Formative Assessment

Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLES

1 **a.** Use a table of values to graph $y = x^2 - x - 2$.

x	y
−2	4
−1	0
0	−2
1	−2
2	0
3	4



b. What are the domain and range of this function? domain: all real numbers; range: $\{y \mid y \geq -2\frac{1}{4}\}$

2 **ARCHERY** The equation $y = -x^2 + 6x + 4$ represents the height, y, of an arrow x seconds after it is shot into the air.

a. Use a table of values to graph $y = -x^2 + 6x + 4$. See bottom margin for graph.

x	y
−1	−3
0	4
1	9
2	12
3	13
4	12
5	9
6	4
7	−3

b. What are the mathematical domain and range of this function? domain: all real numbers; range: $\{y \mid y \leq 13\}$

(continued on the next page)

CHECK Your Progress

1. Use a table of values to graph $y = x^2 + 3$. What are the domain and range of this function? See Ch. 9 Answer Appendix.

Consider the standard form $y = ax^2 + bx + c$. Notice that the value of a in Example 1 is positive and the curve opens upward. The graph of any quadratic function in which a is positive opens upward. The lowest point, or **minimum**, of this graph is located at (1, −7).

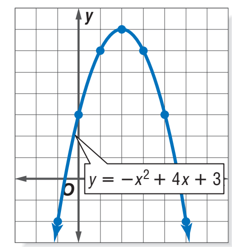
Real-World EXAMPLE — Graph Opens Downward

2 **FLYING DISKS** **The equation $y = -x^2 + 4x + 3$ represents the height y of a flying disk x seconds after it is tossed.**

a. Use a table of values to graph $y = -x^2 + 4x + 3$.

Graph these ordered pairs and connect them with a smooth curve.

x	y
−1	−2
0	3
1	6
2	7
3	6
4	3
5	−2



b. What are the domain and range of this function?

D: $\{x \mid x \text{ is a real number.}\}$

R: $\{y \mid y \leq 7\}$

c. Describe reasonable domain and range values for this situation.

The flying disk is in the air for about 4.6 seconds, so a reasonable domain is $\{x \mid 0 \leq x \leq 4.6\}$. The height of the flying disk ranges from 0 to 7 feet, so a reasonable range is $\{y \mid 0 \leq y \leq 7\}$.

CHECK Your Progress

2. Use a table of values to graph $y = -2x^2 + x + 1$. What are the domain and range of this function? See Ch. 9 Answer Appendix.

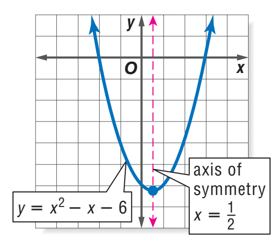
Reading Math

Vertex The plural of vertex is *vertices*. In math, vertex has several meanings. For example, there are the vertex of an angle, the vertices of a polygon, and the vertex of a parabola.

Notice that the value of a in Example 2 is negative and the curve opens downward. The graph of any quadratic function in which a is negative opens downward. The highest point, or **maximum**, of the graph is located at (2, 3). The maximum or minimum point of a parabola is called the **vertex**.

Symmetry and Vertices Parabolas possess a geometric property called **symmetry**. Symmetrical figures are those in which each half of the figure matches the other exactly.

The line that divides a parabola into two halves is called the **axis of symmetry**. Each point on the parabola that is on one side of the axis of symmetry has a corresponding point on the parabola on the other side of the axis. The vertex is the only point on the parabola that is on the axis of symmetry. Notice the relationship between the values a and b and the equation of the axis of symmetry.

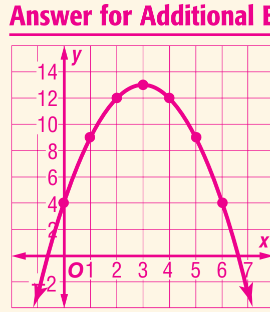


Tips for New Teachers

Preventing Errors

Tell students that their sketches of parabolas do not have to be perfect. However, students should not "connect the dots" with straight lines. The important thing is for the curve to pass through the graphed ordered pairs.

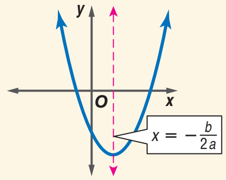
Answer for Additional Example 2a



KEY CONCEPT — Axis of Symmetry of a Parabola

Words The equation of the axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

Model



EXAMPLE Vertex and Axis of Symmetry

3 **Consider the graph of $y = -3x^2 - 6x + 4$.**

a. Write the equation of the axis of symmetry.

In $y = -3x^2 - 6x + 4$, $a = -3$ and $b = -6$.

$x = -\frac{b}{2a}$ Equation for the axis of symmetry of a parabola

$x = -\frac{-6}{2(-3)}$ or -1 $a = -3$ and $b = -6$

The equation of the axis of symmetry is $x = -1$.

b. Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x = -1$ and the vertex lies on the axis, the x-coordinate for the vertex is -1.

$y = -3x^2 - 6x + 4$ Original equation

$y = -3(-1)^2 - 6(-1) + 4$ $x = -1$

$y = -3 + 6 + 4$ Simplify.

$y = 7$ Add.

The vertex is at $(-1, 7)$.

c. Identify the vertex as a maximum or minimum.

Since the coefficient of the x^2 term is negative, the parabola opens downward and the vertex is a maximum point.

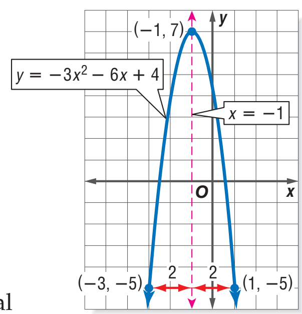
d. Graph the function.

You can use the symmetry of the parabola to help you draw its graph. On a coordinate plane, graph the vertex and the axis of symmetry. Choose a value for x other than -1. For example, choose 1 and find the y-coordinate that satisfies the equation.

$y = -3x^2 - 6x + 4$ Original equation

$y = -3(1)^2 - 6(1) + 4$ Let $x = 1$.

$y = -5$ Simplify.



Graph $(1, -5)$. Since the graph is symmetrical about its axis of symmetry $x = -1$, you can find another point on the other side of the axis of symmetry. The point at $(1, -5)$ is 2 units to the right of the axis. Go 2 units to the left of the axis and plot the point $(-3, -5)$. Repeat this for several other points. Then sketch the parabola.

Study Tip

Coordinates of Vertex

Notice that you can find the x-coordinate by knowing the axis of symmetry. However, to find the y-coordinate, you must substitute the value of x into the quadratic equation.

ADDITIONAL EXAMPLE

2 **c.** Describe reasonable domain and range values for this situation. The arrow is in the air for about 6.6 seconds, so a reasonable domain is $\{x \mid 0 \leq x \leq 6.6\}$. A reasonable range is $\{y \mid 0 \leq y \leq 13\}$.

Additional Examples are also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Symmetry and Vertices

Example 3 shows how to find an equation for the axis of symmetry and the coordinates of the vertex of a parabola. **Example 4** shows how to match an equation with its corresponding graph.

ADDITIONAL EXAMPLE

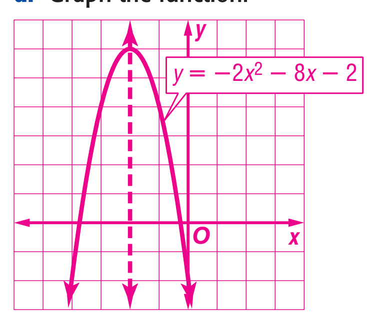
3 Consider the graph of $y = -2x^2 - 8x - 2$.

a. Write the equation of the axis of symmetry. $x = -2$

b. Find the coordinates of the vertex. The coordinates of the vertex are $(-2, 6)$.

c. Identify the vertex as a maximum or minimum. Since the coefficient of the x^2 term is negative, the parabola opens downward and the vertex is a maximum point.

d. Graph the function.



Focus on Mathematical Content

Vertex The maximum or minimum point of a parabola is called the *vertex*. When a quadratic function is written in standard form, $y = ax^2 + bx + c$, and a is positive, the parabola opens upward, and the vertex is a minimum. When a is negative, the parabola opens downward, and the vertex is a maximum.

Intervention

Symmetry and Graphing When students use symmetry to graph parabolas, they only need to find a few points and then reflect those points across the line of symmetry. You may want to suggest that students occasionally check their reflected points by substituting them into the original equation.

ADDITIONAL EXAMPLE

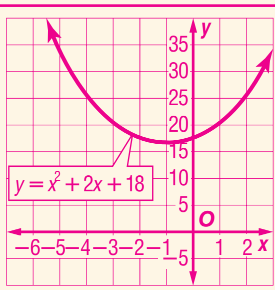
4 **STANDARDS EXAMPLE** Using the item choices from Example 4, which graph corresponds to the graph of $y = -x^2 - 2x - 2$? **D**

Tips for New Teachers — Preventing Errors

Point out that when the answer choices for a test question are graphs, students should look for obvious clues that would make a graph incorrect. Tell them that in Example 4, since the coefficient of the x^2 term is positive, the parabola opens upward, so they can eliminate two of the parabolas that open in the wrong direction. Encourage students to think in terms of the parent graph $y = x^2$ when analyzing graphs of quadratic functions.

Additional Answer

3D.



CHECK Your Progress

Consider the graph of $y = x^2 + 2x + 18$.

3A. Write the equation of the axis of symmetry. $x = -1$

3B. Find the coordinates of the vertex. $(-1, 17)$

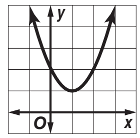
3C. Identify the vertex as a maximum or minimum. **minimum**

3D. Graph the function. **See margin.**

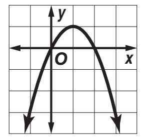
STANDARDS EXAMPLE — Match Equations and Graphs 21.0

4 Which is the graph of $y + 1 = (x + 1)^2$?

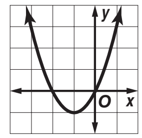
A



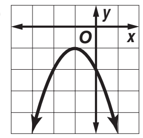
B



C



D



Test-Taking Tip

Substituting Values The ordered pair (0, 0) satisfies the equation $y + 1 = (x + 1)^2$. Since the point at (0, 0) is on the graph, choices A and D can be eliminated.

Read the Item

You are given a quadratic function, and you are asked to choose its graph.

Solve the Item

Step 1 First write the equation in standard form.

$y + 1 = (x + 1)^2$	Original equation
$y + 1 = x^2 + 2x + 1$	$(x + 1)^2 = x^2 + 2x + 1$
$y + 1 - 1 = x^2 + 2x + 1 - 1$	Subtract 1 from each side.
$y = x^2 + 2x$	Simplify.

Step 2 Then find the axis of symmetry of the graph of $y = x^2 + 2x$.

$x = -\frac{b}{2a}$	Equation for the axis of symmetry
$x = -\frac{2}{2(1)}$ or -1	$a = 1$ and $b = 2$

The axis of symmetry is $x = -1$. Look at the graphs. Since only choices C and D have $x = -1$ as their axis of symmetry, you can eliminate choices A and B. Since the coefficient of the x^2 term is positive, the graph opens upward. Eliminate choice D. The answer is C.

CHECK Your Progress

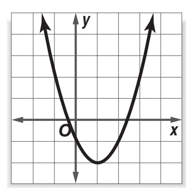
4. Which is the equation of the graph? **H**

F $y - 1 = (x + 2)^2$

G $y - 1 = (x - 2)^2$

H $y + 2 = (x - 1)^2$

J $y - 2 = (x + 1)^2$



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Pre-AP Activity — Use as an Extension

Tell students that the rocket described at the beginning of the lesson explodes 4 seconds after launch at a height of 80 meters. A second rocket, also designed to explode at its highest point, is represented by the equation $h = -5.7t^2 + 34.2t + 1.6$, where h is the rocket's height after t seconds. At what height will the second rocket explode? The second rocket will explode after 3 seconds at approximately 53 meters. How can the two launches be synchronized so that both rockets explode at the same time? Launch the second rocket 1 second after the first.

★ indicates multi-step problem

CHECK Your Understanding

Examples 1, 2 (pp. 471–472)

Use a table of values to graph each function. **1–4. See margin.**

1. $y = x^2 - 5$ **2.** $y = x^2 + 2$

3. $y = -x^2 + 4x + 5$ **4.** $y = x^2 + x - 1$

Example 3 (pp. 473–474)

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. **5–8. See Ch. 9 Answer Appendix for graphs.**

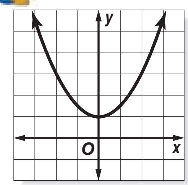
5. $y = x^2 + 4x - 9$ **6.** $y = -x^2 + 5x + 6$

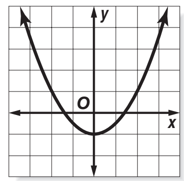
7. $y = -(x - 2)^2 + 1$ **8.** $y = (x + 3)^2 - 4$

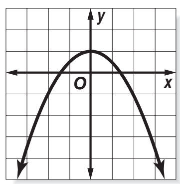
5. $x = -2$; $(-2, -13)$; min
6. $x = 2.5$; $(2.5, 12.25)$; max
7. $x = 2$; $(2, 1)$; max
8. $x = -3$; $(-3, 4)$; min

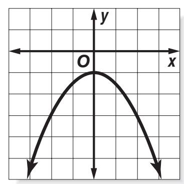
Example 4 (p. 474) **21.0**

9. **STANDARDS PRACTICE** Which is the graph of $y = -\frac{1}{2}x^2 + 1$? **B**

A


C


B


D


Exercises

HOMEWORK HELP	
For Exercises	See Examples
10–15	1, 2
16–29	3

Use a table of values to graph each function. **10–15. See Ch. 9 Answer Appendix.**

10. $y = x^2 - 3$ **11.** $y = -x^2 + 7$

12. $y = x^2 - 2x - 8$ **13.** $y = x^2 - 4x + 3$

14. $y = -3x^2 - 6x + 4$ **15.** $y = -3x^2 + 6x + 1$

Exercise Levels
A: 10–29
B: 30–41
C: 42–46

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. **16–23. See Ch. 9 Answer Appendix for graphs.**

16. $y = 4x^2$ **$x = 0$; $(0, 0)$; min** **17.** $y = -2x^2$ **$x = 0$; $(0, 0)$; max**

18. $y = x^2 + 2$ **$x = 0$; $(0, 2)$; min** **19.** $y = -x^2 + 5$ **$x = 0$; $(0, 5)$; max**

20. $y = -x^2 + 2x + 3$ **$x = 1$; $(1, 4)$; max** **21.** $y = -x^2 - 6x + 15$ **$x = -3$; $(-3, 24)$; max**

22. $y = 3x^2 - 6x + 4$ **$x = 1$; $(1, 1)$; min** **23.** $y = 9 - 8x + 2x^2$ **$x = 2$; $(2, 1)$; min**

24. What is the equation of the axis of symmetry of the graph of $y = -3x^2 + 2x - 5$? **$x = \frac{1}{3}$**

25. Find the equation of the axis of symmetry of the graph of $y = 4x^2 - 5x + 16$.

25. $x = \frac{5}{8}$

DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	10–29, 42–48, 50–65	11–29 odd, 51, 52	10–28 even, 42–48, 50, 53–65
OL Core	11–33 odd, 34–48, 50–65	10–29 odd, 51, 52	30–48, 50, 53–65
AL Advanced /Pre-AP	30–62 (optional: 63–65)		

3 Practice

Formative Assessment

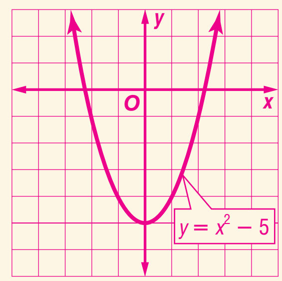
Use Exercises 1–9 to check for understanding.

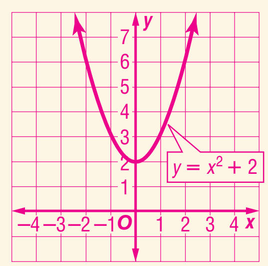
Use the chart at the bottom of this page to customize assignments for your students.

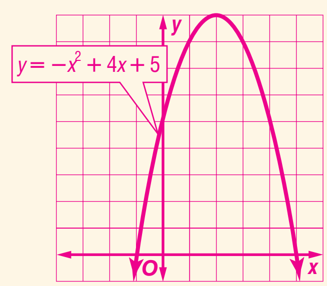
Odd/Even Assignments

Exercises 10–29 and 47–48 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Additional Answers

1.


2.


3.


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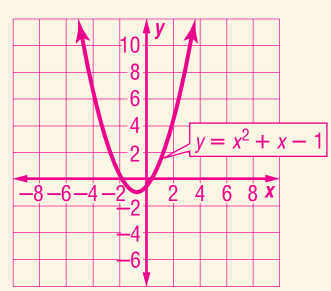
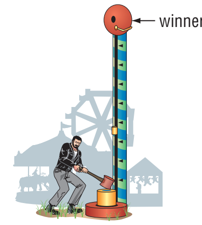


Lesson 9-1 Graphing Quadratic Functions **475**

ENTERTAINMENT For Exercises 26 and 27, use the following information.



A carnival game involves striking a lever that forces a weight up a tube. If the weight reaches 20 feet to ring the bell, the contestant wins a prize. The equation $h = -16t^2 + 32t + 3$ gives the height of the weight if the initial velocity is 32 feet per second.

★ **26.** Find the maximum height of the weight. **19 ft**

27. Will a prize be won? Explain. **No, the height needs to be 20 ft to win a prize.**

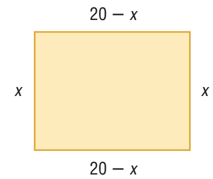
Real-World Link

The Gateway Arch is part of a tribute to Thomas Jefferson, the Louisiana Purchase, and the pioneers who settled the West. Each year about 2.5 million people visit the arch.

Source: *World Book Encyclopedia*

PETS For Exercises 28 and 29, use the following information.

Miriam has 40 meters of fencing to build a pen for her dog.



★ **28.** Use the diagram to write an equation for the area A of the pen. Describe a reasonable domain and range for this situation. **See Ch. 9 Answer Appendix.**

★ **29.** What value of x will result in the greatest area? What is the greatest possible area of the pen? **10 m; 100 m²**

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. **30–33. See Ch. 9 Answer Appendix for graphs.**

30. $y = -2(x - 4)^2 - 3$

31. $y + 2 = x^2 - 10x + 25$

32. $y - 5 = \frac{1}{3}(x + 2)^2$

33. $y + 1 = \frac{1}{3}(x + 1)^2$

30. $x = 4$; $(4, -3)$; max

31. $x = 5$; $(5, -2)$; min

32. $x = -2$; $(-2, 5)$; min

33. $x = -1$; $(-1, -1)$; min

34. The vertex of a parabola is at $(-4, -3)$. If one x-intercept is -11, what is the other x-intercept? **3**

35. What is the equation of the axis of symmetry of a parabola if its x-intercepts are -6 and 4? **$x = -1$**

ARCHITECTURE For Exercises 36–38, use the following information.

The shape of the Gateway Arch in St. Louis, Missouri, is a *catenary* curve. It resembles a parabola with the equation $h = -0.00635x^2 + 4.0005x - 0.07875$, where h is the height in feet and x is the distance from one base in feet.

36. What is the equation of the axis of symmetry? **$x = 315$**

37. What is the distance from one end of the arch to the other? **630 ft**

38. What is the maximum height of the arch? **630 ft**

FOOTBALL For Exercises 39–41, use the following information.

A football is kicked from ground level at an initial velocity of 90 feet per second. The equation $h = -16t^2 + 90t$ gives the height h of the football after t seconds.

39. What is the height of the ball after one second? **74 ft**

40. When is the ball 126 feet high? **3 s or 2.625 s**

41. When is the height of the ball zero feet? Describe the events these represent. **$t = 0$ and 5.625 s; The height of the ball is zero feet before it is kicked and again when the ball lands on the ground.**

EXTRA PRACTICE
See pages 735, 752.
Math Online
Self-Check Quiz at ca.algebra1.com

H.O.T. Problems

42. OPEN ENDED Sketch a parabola that models a real-life situation and describe what the vertex represents. Determine reasonable domain and range values for this type of situation. **See Ch. 9 Answer Appendix.**

43. REASONING Sketch the parent graph of the function $y = 3x^2 - 5x - 2$. **See Ch. 9 Answer Appendix.**

Study Guide and Intervention
CRM pp. 6–7 OL AL ELL

9-1 Study Guide and Intervention
Graphing Quadratic Functions

Graph Quadratic Functions

Quadratic Function	a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$	Example: $y = 2x^2 + 3x + 8$

The parent graph of the family of quadratic fuctions is $y = x^2$. Graphs of quadratic functions have a general shape called a **parabola**. A parabola opens upward and has a **minimum point** when the value of a is positive, and a parabola opens downward and has a **maximum point** when the value of a is negative.

Example 1 a. Use a table of values to graph $y = x^2 - 4x + 1$.

x	y
−1	6
0	1
1	−2
2	−3
3	−2
4	1

Graph the ordered pairs in the table and connect them with a smooth curve.

b. What is the domain and range of this function? D: {x | x is a real number.} R: {y | y ≥ −3}

Example 2 a. Use a table of values to graph $y = -x^2 - 6x - 7$.

x	y
−6	−7
−5	−2
−4	1
−3	2
−2	1
−1	−2
0	−7

Graph the ordered pairs in the table and connect them with a smooth curve.

b. What is the domain and range of this function? D: {x | x is a real number.} R: {y | y ≤ 2}

Exercises

Use a table of values to graph each function. Determine the domain and range.

1. $y = x^2 + 2$ D: {x | x is a real number.} R: {y | y ≥ 2}
2. $y = -x^2 - 4$ D: {x | x is a real number.} R: {y | y ≤ −4}
3. $y = x^2 - 3x + 2$ D: {x | x is a real number.} R: {y | y ≥ −1/4}

Chapter 9 6 Glencoe Algebra 1

Practice
CRM p. 9 OL AL

9-1 Practice
Graphing Quadratic Functions

Use a table of values to graph each function. Determine the domain and range.

1. $y = -x^2 + 2$ D: {x | x is a real number.} R: {y | y ≤ 2}
2. $y = x^2 - 6x + 3$ D: {x | x is a real number.} R: {y | y ≥ −6}
3. $y = -2x^2 - 8x - 5$ D: {x | x is a real number.} R: {y | y ≤ 3}

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

4. $y = -x^2 + 3$ x = 0; (0, 3); max
5. $y = -2x^2 + 8x - 3$ x = 2; (2, 5); max
6. $y = 2x^2 + 8x + 1$ x = −2; (−2, −7); min

PHYSICS For Exercises 7–9, use the following information.
Miranda throws a set of keys up to her brother, who is standing on a balcony 38 feet above the ground. The equation $h = -16t^2 + 40t + 5$ gives the height h of the keys after t seconds.

7. How long does it take the keys to reach their highest point? 1.25 s
8. How high do the keys reach? 30 ft
9. Will her brother be able to catch the keys? Explain. No, the keys will be 8 ft short of their target.
10. What is a reasonable domain and range for this problem? D: {x | x ≥ 0} R: {y | 0 ≤ y ≤ 30}

BASEBALL For Exercises 11–13, use the following information.
A player hits a baseball into the outfield. The equation $h = -0.005x^2 + x + 3$ gives the path of the ball, where h is the height and x is the horizontal distance the ball travels.

11. What is the equation of the axis of symmetry? x = 100
12. What is the maximum height reached by the baseball? 53 ft
13. An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it? 200 ft

Chapter 9 9 Glencoe Algebra 1

Word Problem Practice
CRM p. 10 OL AL

9-1 Word Problem Practice
Graphing Quadratic Functions

1. OLYMPICS Olympics were held in 1896 and have been held every four years (except 1916, 1940, and 1944). The winning height y in men's pole vault at any number Olympiad x can be approximated by the equation $y = 0.37x^2 + 4.3x + 126$. Complete the table to estimate the pole vault heights in each of the Olympic Games. Round your answers to the nearest tenth.

Year	Olympiad (x)	Height (y inches)
1896	1	130.7
1900	2	136.1
1924	7	174.2
1936	10	206.0
1964	15	273.8
2008	26	487.9

2. PHYSICS Mrs. Capwell's physics class investigates what happens when a ball is given an initial push, rolls up, and then back down an inclined plane. The class finds that $y = -x^2 + 6x$ accurately predicts the ball's position y after rolling x seconds. On the graph of the equation, what would be the y value when $x = 4$? 8

3. ARCHITECTURE A hotel's main entrance is in the shape of a parabolic arch. The equation $y = -x^2 + 10x$ models the arch height y for any distance x from one side of the arch. Use a graph to determine its maximum height. 25 ft

4. SOFTBALL Olympic softball gold medalist Michele Smith pitches a curveball with a speed of 64 feet per second. If she throws the ball straight upward at this speed, the ball's height h (in feet) after t seconds is given by $h = -16t^2 + 64t$. Find the coordinates of the vertex of the graph of the ball's height and interpret its meaning. (2, 64); After 2 seconds, the ball reaches its highest point, 64 ft above the ground.

GEOMETRY For Exercises 5–7, use the following information.
Teddy is building the rectangular deck shown below.

5. Write the equation representing the area of the deck. $y = (x - 2)(x + 6)$ or $y = x^2 + 4x - 12$
6. What is the equation of the axis of symmetry? $x = -2$
7. Graph the equation and label its vertex.

Chapter 9 10 Glencoe Algebra 1

Enrichment
CRM p. 11 OL AL

9-1 Enrichment

Translating Quadratic Graphs

When a figure is moved to a new position without undergoing any rotation, then the figure is said to have been **translated** to that position.

The graph of a quadratic equation in the form $y = (x - b)^2 + c$ is a translation of the graph of $y = x^2$.

Start with $y = x^2$.
Slide to the right 4 units.
$y = (x - 4)^2$
Then slide up 3 units.
$y = (x - 4)^2 + 3$

BL = Below Grade Level
OL = On Grade Level
AL = Above Grade Level
ELL = English Language Learner

Additional pages not shown:

CRM *Lesson Reading Guide*, p. 5 BL OL ELL
CRM *Skills Practice*, p. 8 BL OL

REASONING Let $f(x) = x^2 - 9$.

44. What is the domain of $f(x)$? **all real numbers**

45. What is the range of $f(x)$? **$\{y \mid y \geq -9\}$**

46. For what values of x is $f(x)$ negative? **$\{x \mid -3 < x < 3\}$**

47. When x is a real number, what are the domain and range of $f(x) = \sqrt{x^2 - 9}$?

47. domain $x \leq -3$ and $x \geq 3$; range $y \geq 0$

48. REASONING Determine the range of $f(x) = (x - 5)^2 - 6$. **$\{y \mid y \geq -6\}$**

49. CHALLENGE Write and graph a quadratic equation whose graph has the axis of symmetry $x = -\frac{3}{8}$. Summarize the steps that you took to determine the equation. **See margin.**

50. *Writing in Math* Use the information about a rocket's path on page 471 to explain how a fireworks display can be coordinated with recorded music. Include an explanation of how to determine when the rocket will explode and how to find the height of the rocket when it explodes. **See margin.**

STANDARDS PRACTICE 6NS1.4

51. In the graph of the function $y = x^2 - 3$, which describes the shift in the vertex of the parabola if, in the function, -3 is changed to 1? **B**

A 2 units up

B 4 units up

C 2 units down

D 4 units down

52. REVIEW The costs of two packs of Brand A gum and two packs of Brand B gum are shown in the table. What percent of the cost of Brand B gum does James save by buying two packs of Brand A gum? **G**

Gum	Cost of Two Packs
Brand A	\$1.98
Brand B	\$2.50

F 11.6% H 26.3%

G 20.8% J 79.2%

Spiral Review

Factor each polynomial, if possible. (Lessons 8-5 and 8-6)

53. $x^2 + 6x - 9$ **prime**

54. $a^2 + 22a + 121$ **$(a + 11)^2$**

55. $4m^2 - 4m + 1$ **$(2m - 1)^2$**

56. $4q^2 - 9$ **$(2q - 3)(2q + 3)$**

57. $2a^2 - 25$ **prime**

58. $1 - 16g^2$ **$(1 - 4g)(1 + 4g)$**

Find each sum or difference. (Lesson 7-5)

59. $(13x + 9y) + 11y$ **$13x + 20y$**

60. $(8 - 2c^2) + (1 + c^2)$ **$9 - c^2$**

61. $(7p^2 - p - 7) - (p^2 + 11)$ **$6p^2 - p - 18$**

62. RECREATION At a recreation facility, 3 members and 3 nonmembers pay a total of \$180 to take an aerobics class. A group of 5 members and 3 nonmembers pay \$210 to take the same class. How much does it cost each to take an aerobics class? (Lesson 5-3) **\$15 for members, \$45 for nonmembers**

GET READY for the Next Lesson

PREREQUISITE SKILL Find the x-intercept of the graph of each equation. (Lesson 3-3)

63. $3x + 4y = 24$ **8**

64. $2x - 5y = 14$ **7**

65. $-2x - 4y = 7$ **-3.5**

Tips for New Teachers — Student Misconceptions

Point out that although the equations in Exercises 30–33 are not in the standard form $y = ax^2 + bx + c$, they still have a degree of 2, so they are quadratic. Furthermore, when the expressions in parentheses are multiplied, the equations take on more traditional forms.

4 Assess

Ticket Out the Door Make several copies each of five graphs of parabolas. Give one graph to each student. As the students leave the room, ask them to tell you the coordinates of the parabolas' vertices and to identify them as maximums or minimums.

FOLDABLES Study Organizer — Foldables™ Follow-Up

At the end of this lesson, remind students to take notes and show examples on the page in their Foldables labeled 9-1.

49. Sample answer: $y = 4x^2 + 3x + 5$; write the equation for the axis of symmetry of a parabola, $x = -\frac{b}{2a}$. From the equation, $b = 3$ and $2a = 8$, so $a = 4$. Substitute these values for a and b into the equation $y = ax^2 + bx + c$.

50. Sample answer: In order to coordinate a firework with recorded music, you must know when and how high it will explode. The rocket will explode when the rocket reaches the vertex or when $t = -\frac{39.2}{2(-4.9)}$, which is 4 seconds. The height of the rocket when it explodes is the height when $t = 4$. Therefore, $h = -4.9(4^2) + 39.2(4) + 1.6$ or 80 meters

EXTEND 9-1 Lesson Notes

1 Focus

Objective Use the graphing calculator to graph families of quadratic graphs.

Materials

- graphing calculator

Teaching Tip Remind students that the $\boxed{x^2}$ key squares a quantity but does not enter x^2 into the equation. To enter $3x^2$, press 3 $\boxed{X,T,\theta,n}$ $\boxed{x^2}$.

2 Teach

Working in Cooperative Groups Put students in groups of 2 or 3, mixing abilities. Have groups complete Activities 1 and 2 and Exercises 1–4.

- Make sure students know how to set the standard viewing window for graphs and how to change the appearance of the viewing window.
- To help students remember how the value of a in $y = ax^2$ affects the shape of the graph, suggest that students sketch some of the parabolas and their equations on grid paper.
- Have students use the TRACE feature to help them identify the different parabolas on the screen. After pressing $\boxed{\text{TRACE}}$, students can use the arrow keys to move the cursor. The up and down arrows switch the cursor between graphs. The left and right arrows move the cursor along the individual graphs. The graph on which the cursor lies is identified in the top left-hand corner of the screen.

Practice Have students complete Exercises 5–12.

EXTEND 9-1 Graphing Calculator Lab
The Family of Quadratic Functions

Standard 21.0 **Students graph quadratic functions** and know that their roots are the x-intercepts.

The parent function of the family of quadratic functions is $y = x^2$.

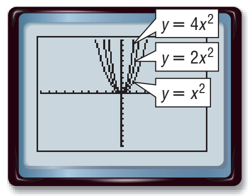
Concepts in Motion Animation ca.algebra1.com

ACTIVITY 1

Graph each group of equations on the same screen. Use the standard viewing window. Compare and contrast the graphs.

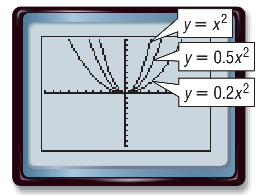
KEYSTROKES: *Review graphing equations on pages* 162 *and* 163.

a. $y = x^2, y = 2x^2, y = 4x^2$



Each graph opens upward and has its vertex at the origin. The graphs of $y = 2x^2$ and $y = 4x^2$ are narrower than the graph of $y = x^2$.

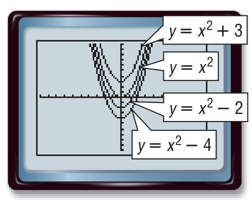
b. $y = x^2, y = 0.5x^2, y = 0.2x^2$



Each graph opens upward and has its vertex at the origin. The graphs of $y = 0.5x^2$ and $y = 0.2x^2$ are wider than the graph of $y = x^2$.

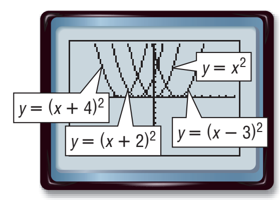
1A. How does the value of a in $y = ax^2$ affect the shape of the graph? **It makes the graph narrower or wider.**

c. $y = x^2, y = x^2 + 3, y = x^2 - 2, y = x^2 - 4$



Each graph opens upward and has the same shape as $y = x^2$. However, each parabola has a different vertex, located along the y-axis.

d. $y = x^2, y = (x - 3)^2, y = (x + 2)^2, y = (x + 4)^2$



Each graph opens upward and has the same shape as $y = x^2$. However, each parabola has a different vertex located along the x-axis.

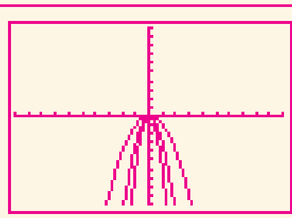
1B. How does the value of the constant affect the position of the graph? **It moves the graph up and down.**

1C. How is the location of the vertex related to the equation of the graph? **1C. The vertex moves right or left depending on the value in the parentheses.**

Math Online **Other Calculator Keystrokes at** ca.algebra1.com

Additional Answer

1.



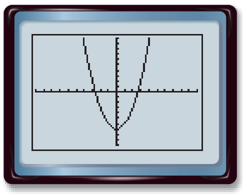
All of the graphs open downward from the origin. $y = -3x^2$ is narrower than $y = -x^2$, and $y = -6x^2$ is the narrowest.

Suppose you graph the same equation using different windows. How will the appearance of the graph change?

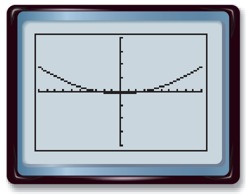
ACTIVITY 2

Graph $y = x^2 - 7$ in each viewing window. What conclusions can you draw about the appearance of a graph in the window used?

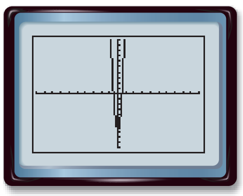
a. **standard viewing window**



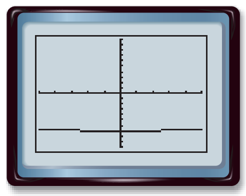
b. **$[-10, 10]$ scl: 1 by $[-200, 200]$ scl: 50**



c. **$[-50, 50]$ scl: 5 by $[-10, 10]$ scl: 1**



d. **$[-0.5, 0.5]$ scl: 0.1 by $[-10, 10]$ scl: 1**



Without knowing the window, graph **b** might be of the family $y = ax^2$, where $0 < a < 1$. Graph **c** looks like a member of $y = ax^2 - 7$, where $a > 1$. Graph **d** looks more like a line. However, all are graphs of the same equation.

Exercises

Graph each family of equations on the same screen. Compare and contrast the graphs. 1–4. See margin.

1. $y = -x^2$, $y = -3x^2$, $y = -6x^2$

2. $y = -x^2$, $y = -0.6x^2$, $y = -0.4x^2$

3. $y = -x^2$, $y = -(x + 5)^2$, $y = -(x - 4)^2$

4. $y = -x^2$, $y = -x^2 + 7$, $y = -x^2 - 5$

Use the graphs on page 478 and Exercises 1–4 above to predict the appearance of the graph of each equation. Then draw the graph. 5–8. See Ch. 9 Answer Appendix.

5. $y = -0.1x^2$ **6.** $y = (x + 1)^2$ **7.** $y = 4x^2$ **8.** $y = x^2 - 6$

Describe how each change in $y = x^2$ would affect the graph of $y = x^2$. Be sure to consider all values of a, h, and k. 9–12. See margin.

9. $y = ax^2$ **10.** $y = (x + h)^2$ **11.** $y = x^2 + k$ **12.** $y = (x + h)^2 + k$

10. The graph has the same shape as $y = x^2$, but is shifted h units (left if $h > 0$, right if $h < 0$).

11. The graph has the same shape as $y = x^2$, but is shifted k units (up if $k > 0$, down if $k < 0$).

12. The graph has the same shape as $y = x^2$, but is shifted h units left or right and k units up or down as prescribed in Exercises 10 and 11.

3 Assess

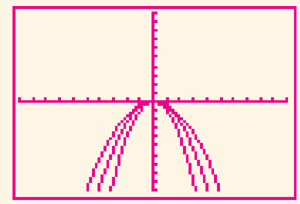
Formative Assessment

Use Exercise 3 to assess students' ability to use the TRACE function to identify the vertex and x-intercept(s) of each function in the list.

From Concrete to Abstract Ask students to explain how the functions in Exercises 9–12 differ from the function $y = x^2$. They should consider different values of a, h, and k.

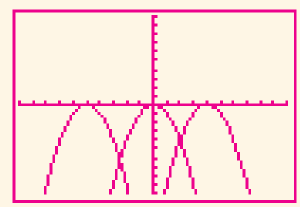
Additional Answers

2.



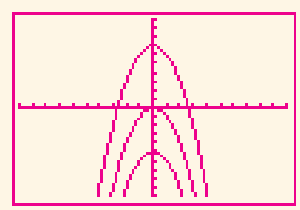
All of the graphs open downward from the origin. $y = -0.6x^2$ is wider than $y = -x^2$, and $y = -0.4x^2$ is the widest.

3.



All of the graphs open downward, have the same shape, and have vertices along the x-axis. However, each vertex is different.

4.



All of the graphs open downward, have the same shape, and have vertices along the y-axis. However each vertex is different.

9. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $0 < |a| < 1$, the graph is wider than the graph of $y = x^2$. If $a < 0$, it opens downward. If $a > 0$, it opens upward.

9-2 Lesson Notes

1 Focus

Standards Alignment

Before Lesson 9-2
Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems from Standard 7AF3

Lesson 9-2
Graph quadratic functions and know that their roots are the x- intercepts and determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points from Standards 22.0 and 21.0

After Lesson 9-2
Solve and graph quadratic equations and quadratic equations in the complex number system from Standard 2A8.0

2 Teach

Scaffolding Questions

Have students read *Get Ready for the Lesson.*

Ask:

- If one of the x-intercepts represents where the ball hits the ground, what represents the ground? the x-axis
- Suppose the green is uphill from the tee. How would this affect the value of the y-coordinate of the spot where the ball lands? The y-coordinate would be positive if the shot was uphill.

9-2 Solving Quadratic Equations by Graphing

Main Idea

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

Standard 21.0 Students graph quadratic functions and know that their roots are the x-intercepts.

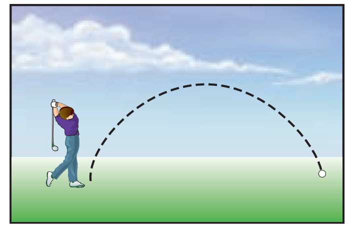
Standard 22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.

New Vocabulary

quadratic equation
roots
zeros
double root

GET READY for the Lesson

A golf ball follows a path much like a parabola. Because of this property, quadratic functions can be used to simulate parts of a computer golf game. One of the x-intercepts of the quadratic function represents the location where the ball will hit the ground.



Solve by Graphing A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. The value of the related quadratic function is 0.

Quadratic Equation	Related Quadratic Function
$x^2 - 2x - 3 = 0$	$f(x) = x^2 - 2x - 3$

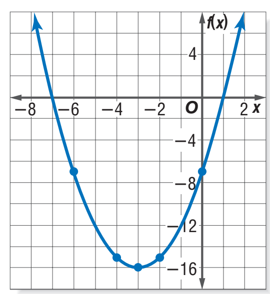
The solutions of a quadratic equation are called the **roots** of the equation. The roots of a quadratic equation can be found by finding the x-intercepts or **zeros** of the related quadratic function.

EXAMPLE Two Roots

1 **Solve $x^2 + 6x - 7 = 0$ by graphing.**

Graph the related function $f(x) = x^2 + 6x - 7$. The equation of the axis of symmetry is $x = -\frac{6}{2(1)}$ or $x = -3$. When x equals -3, $f(x)$ equals $(-3)^2 + 6(-3) - 7$ or -16. So, the coordinates of the vertex are $(-3, -16)$. Make a table of values to find other points to sketch the graph.

x	$f(x)$
−8	9
−6	−7
−4	−15
−3	−16
−2	−15
0	−7
2	9



To solve $x^2 + 6x - 7 = 0$, you need to know where the value of $f(x)$ is 0. This occurs at the x-intercepts. The x-intercepts of the parabola appear to be −7 and 1.

Concepts in Motion
Animation ca.algebra1.com

Study Tip

***x*-intercepts**
The x-intercepts of a graph are also called the *horizontal intercepts.*

Lesson 9-2 Resources

Chapter 9 Resource Masters

Lesson Reading Guide, p. 12 BL OL ELL
Study Guide and Intervention, pp. 13–14 BL OL ELL
Skills Practice, p. 15 BL OL
Practice, p. 16 OL AL
Word Problem Practice, p. 17 OL AL
Enrichment, p. 18 OL AL
Quiz 1, p. 51

Transparencies

5-Minute Check Transparency 9-2

Additional Print Resources

Noteables™ Interactive Study Notebook with Foldables™

Technology

ca.algebra1.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

Study Tip

Common Misconception

Although solutions found by graphing may appear to be exact, you cannot be sure that they are exact. Solutions need to be verified by substituting into the equation and checking, or by using the algebraic methods that you will learn in this chapter.

CHECK Solve by factoring.

$x^2 + 6x - 7 = 0$ Original equation

$(x + 7)(x - 1) = 0$ Factor.

$x + 7 = 0$ or $x - 1 = 0$ Zero Product Property

$x = -7$ ✔ $x = 1$ ✔ The solutions are -7 and 1.

CHECK Your Progress

1. Solve $-c^2 + 5c - 4 = 0$ by graphing.

1, 4; See Ch. 9 Answer Appendix for graph.

Quadratic equations always have two roots. However, these roots are not always two distinct numbers. Sometimes the two roots are the same number, called a **double root**. In other cases the roots are not real numbers.

EXAMPLE A Double Root

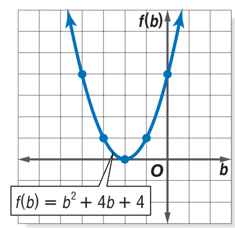
2 **Solve $b^2 + 4b = -4$ by graphing.**

First rewrite the equation so one side is equal to zero.

$b^2 + 4b = -4$ Original equation

$b^2 + 4b + 4 = 0$ Add 4 to each side.

Graph the related function $f(b) = b^2 + 4b + 4$.



Notice that the vertex of the parabola is the b-intercept. Thus, one solution is -2. What is the other solution?

Try solving the equation by factoring.

$b^2 + 4b + 4 = 0$ Original equation

$(b + 2)(b + 2) = 0$ Factor.

$b + 2 = 0$ or $b + 2 = 0$ Zero Product Property

$b = -2$ $b = -2$ The solution is -2.

CHECK Your Progress

2. Solve $0 = x^2 - 6x + 9$ by graphing. **3; See Ch. 9 Answer Appendix for graph.**

EXAMPLE No Real Roots

Study Tip

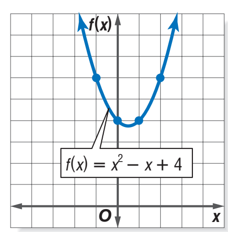
Empty Set

The symbol ∅, indicating an empty set, is often used to represent no real solutions.

3 **Solve $x^2 - x + 4 = 0$ by graphing.**

Graph the related function $f(x) = x^2 - x + 4$.

x	$f(x)$
−1	6
0	4
1	4
2	6



The graph has no x-intercept. Thus, there are no real number solutions for this equation.

CHECK Your Progress

3. Solve $-t^2 - 3t = 5$ by graphing. **∅; See Ch. 9 Answer Appendix for graph.**

Math Online Extra Examples at ca.algebra1.com

Solve By Graphing

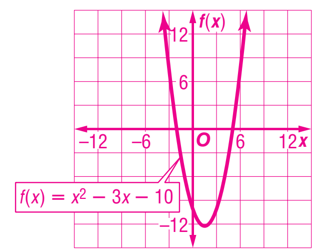
Example 1 shows how to use graphing to find the two roots of a quadratic equation. **Example 2** shows how to use graphing to find the double root of a quadratic equation. **Example 3** shows how to use a graph to identify a quadratic equation that has no real number solutions.

Formative Assessment

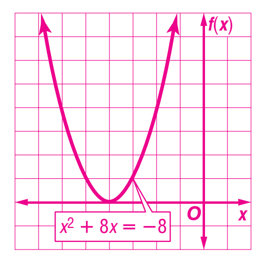
Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLES

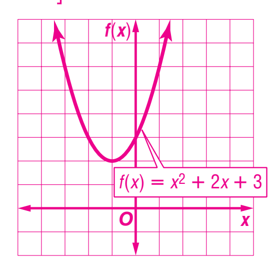
1 Solve $x^2 - 3x - 10 = 0$ by graphing. $\{-2, 5\}$



2 Solve $x^2 + 8x = -16$ by graphing. $\{-4\}$



3 Solve $x^2 + 2x + 3 = 0$ by graphing. ∅ [empty set, or no real roots]



Additional Examples are also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Differentiated Instruction

Logical Learners Students may assume that the vertex of a parabola lies on coordinates that are integers. Point out that in Example 3, the y-value for the vertex of the function is greater than 3 and somewhat less than 4.

Tips for New Teachers — Solving with Graphs and Tables

A graphing calculator is a powerful tool for solving quadratic equations by graphing or using tables. You may wish to discuss these techniques with students.

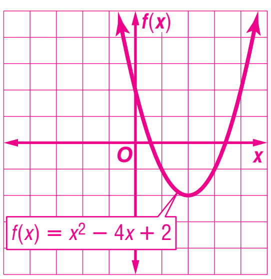
Estimate Solutions

Example 4 shows how to find roots of a quadratic equation by factoring. **Example 5** shows how to use graphing to estimate rational roots of a quadratic equation when integral roots cannot be found. **Example 6** shows how to estimate a solution to a real-world problem involving quadratic equations.

ADDITIONAL EXAMPLES

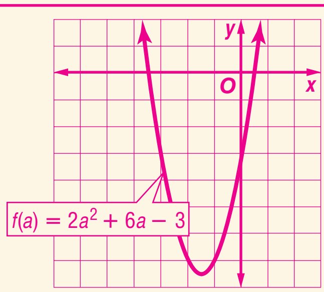
4 Use factoring to determine how many times the graph of $f(x) = x^2 + 3x - 10$ intersects the x-axis. Identify each root. 2; -5, 2

5 Solve $x^2 - 4x + 2 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie. One root is between 0 and 1, and the other root is between 3 and 4.



Additional Answer

4.



Factoring can be used to determine whether the graph of a quadratic function intersects the x-axis in zero, one, or two points.

EXAMPLE Factoring

4 **Use factoring to determine how many times the graph of $f(x) = x^2 + x - 12$ intersects the x-axis. Identify each root.**

The graph intersects the x-axis when $f(x) = 0$.

$x^2 + x - 12 = 0$ Original equation

$(x - 3)(x + 4) = 0$ Factor.

Since the trinomial factors into two distinct factors, the graph of the function intersects the x-axis 2 times. The roots are $x = 3$ and $x = -4$.

CHECK Your Progress

4. Use factoring to determine how many times the graph of $f(x) = x^2 - 10x + 25$ intersects the x-axis. Identify each root. **1; 5**

Estimate Solutions In Examples 1 and 2, the roots of the equation were integers. Usually the roots of a quadratic equation are not integers. In these cases, use estimation to approximate the roots of the equation.

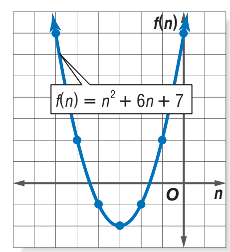
EXAMPLE Rational Roots

5 **Solve $n^2 + 6n + 7 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.**

Graph the related function $f(n) = n^2 + 6n + 7$.

n	$f(n)$
-6	7
-5	2
-4	-1
-3	-2
-2	-1
-1	2
0	7

Notice that the value of the function changes from negative to positive between the n values of -5 and -4 and between -2 and -1.



The n-intercepts are between -5 and -4 and between -2 and -1. So, one root is between -5 and -4, and the other root is between -2 and -1.

Study Tip

Location of Roots

Since quadratic functions are continuous, there must be a zero between x-values when their function values have opposite signs.

CHECK Your Progress

5. Solve $2a^2 + 6a - 3 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie. **$-4 < a < -3$, $0 < a < 1$; See margin for graph.**

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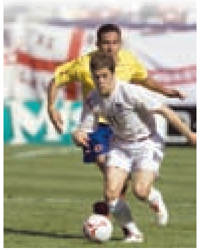
Focus on Mathematical Content

No Real Roots Creating a table of values before graphing a function helps to reveal whether a function has no real roots. If all y-values of a function are positive, first decreasing then increasing, or if all are negative, first increasing then decreasing, the graph of the function does not cross the x-axis, and there are no real roots.

Double Root When there are two identical factors for a quadratic function, there is only one root, called a *double root.* If either the greatest or least y-value in the range of the function is 0, then the vertex is on the x-axis and the solution is a double root.



Real-World Link

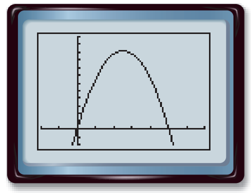
The game of soccer, called "football" in countries other than North America, began in 1857 in Britain. It is played on every continent of the world.

Source: worldsoccer.about.com

Real-World EXAMPLE

6 **SOCCER** **If a goalie kicks a soccer ball with an upward velocity of 65 feet per second and his foot meets the ball 3 feet off the ground, the function $y = -16t^2 + 65t + 3$ represents the height of the ball y in feet after t seconds. Approximately how long is the ball in the air?**

You need to find the solution of the equation $0 = -16t^2 + 65t + 3$. Use a graphing calculator to graph the related function $y = -16t^2 + 65t + 3$. The x-intercept is about 4. Therefore, the ball is in the air about 4 seconds.



[−2, 7] scl: 1 by [−20, 80] scl: 10

CHECK Your Progress

6. **NUMBER THEORY** Use a quadratic equation to find two numbers whose sum is 5 and whose product is −24. **−3, 8**

★ indicates multi-step problem

CHECK Your Understanding

Examples 1–3 (pp. 480–481)

Solve each equation by graphing. **1–3. See margin.**

1. $x^2 - 7x + 6 = 0$ **2.** $-a^2 - 10a = 25$ **3.** $c^2 + 3 = 0$

Example 4 (p. 482)

Use factoring to determine how many times the graph of each function intersects the x-axis. Identify each root.

4. $f(x) = x^2 + 2x - 24$ **2; −6, 4** **5.** $f(x) = x^2 + 14x + 49$ **1; −7**

Example 5 (p. 482)

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

6. $-t^2 - 5t + 1 = 0$ **7.** $0 = x^2 - 16$ **8.** $w^2 - 3w = 5$

6–8. See margin.

Example 6 (p. 483)

9. **NUMBER THEORY** Two numbers have a sum of 4 and a product of −12. Use a quadratic equation to determine the two numbers. **−2, 6**

Exercises

HOMEWORK HELP

For Exercises	See Examples
10–17	1–3
18–21	4
22–30	5
31, 32	6

Exercise Levels
A: 10–32
B: 33–40
C: 41–44

Solve each equation by graphing. **10–15. See margin.**

10. $c^2 - 5c - 24 = 0$ **11.** $5n^2 + 2n + 6 = 0$ **12.** $0 = x^2 + 6x + 9$

13. $-b^2 + 4b = 4$ **14.** $x^2 + 2x + 5 = 0$ **15.** $-2r^2 - 6r = 0$

16. The roots of a quadratic equation are −2 and −6. The minimum point of the graph of its related function is at (−4, −2). Sketch the graph of the function and compare the graph to the graph of the parent function $y = x^2$.

17. The roots of a quadratic equation are −6 and 0. The maximum point of the graph of its related function is at (−3, 4). Sketch the graph of the function and compare the graph to the graph of the parent function $y = x^2$.

16–17. See Ch. 9 Answer Appendix.

Use factoring to determine how many times the graph of each function intersects the x-axis. Identify each root.

18. $g(x) = x^2 - 8x + 16$ **1; 4** **19.** $h(x) = x^2 + 12x + 32$ **2; −4, −8**

20. $f(x) = x^2 + 3x + 4$ **0; no real roots** **21.** $g(x) = x^2 + 3x + 4$ **0; no real roots**

ADDITIONAL EXAMPLE

6 **MODEL ROCKETS** Shelly built a model rocket for her science project. The equation $y = -16t^2 + 250t$ models the flight of the rocket launched from ground level at a velocity of 250 feet per second, where y is the height of the rocket in feet after t seconds. For how many seconds was Shelly's rocket in the air? between 15 and 16 seconds

3 Practice

Formative Assessment

Use Exercises 1–7 to check for understanding.

Use the chart at the bottom of this page to customize your assignments for students.

Odd/Even Assignments

Exercises 8–26 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Additional Answers

See Chapter 9 Answer Appendix for graphs.

1. 1, 6
2. −5
3. ∅
6. $-6 < t < -5$, $0 < t < 1$
7. −4, 4
8. $-2 < w < -1$, $4 < w < 5$
10. −3, 8
11. ∅
12. −3
13. 2
14. ∅
15. −3, 0

DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	10–32, 41, 44–60	11–31 odd, 45, 46	10–32 even, 41, 44, 47–60
OL Core	11–31 odd, 33–41, 44–60	10–32, 45, 46	33–41, 44, 47–60
AL Advanced /Pre-AP	33–56 (optional: 57–60)		

Lesson 9-2 Solving Quadratic Equations by Graphing 483

Study Guide and Intervention
CRM pp. 13–14 OL AL ELL

9-2 Study Guide and Intervention

Solving Quadratic Equations by Graphing

Solve by Graphing

Quadratic Equation	an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$

The solutions of a quadratic equation are called the **roots** of the equation. The roots of a quadratic equation can be found by graphing the related quadratic function $f(x) = ax^2 + bx + c$ and finding the x-intercepts or **zeros** of the function.

Example 1 Solve $x^2 + 4x + 3 = 0$ by graphing.

Graph the related function $f(x) = x^2 + 4x + 3$. The equation of the axis of symmetry is $x = -\frac{4}{2(1)}$ or -2. The vertex is at $(-2, -1)$. Graph the vertex and several other points on either side of the axis of symmetry.

To solve $x^2 + 4x + 3 = 0$, you need to know where the value of $f(x) = 0$. This occurs at the x-intercepts, -3 and -1. The solutions are -3 and -1.

Example 2 Solve $x^2 - 6x + 9 = 0$ by graphing.

Graph the related function $f(x) = x^2 - 6x + 9$. The equation of the axis of symmetry is $x = \frac{6}{2(1)}$ or 3. The vertex is at $(3, 0)$. Graph the vertex and several other points on either side of the axis of symmetry.

To solve $x^2 - 6x + 9 = 0$, you need to know where the value of $f(x) = 0$. The vertex of the parabola is the x-intercept. Thus, the only solution is 3.

Exercises

Solve each equation by graphing.

1. $x^2 + 7x + 12 = 0$ $-3; -4$
2. $x^2 - x - 12 = 0$ $4, -3$
3. $x^2 - 4x + 5 = 0$ no real roots

Practice
CRM p. 16 OL AL

9-2 Practice

Solving Quadratic Equations by Graphing

Solve each equation by graphing.

1. $x^2 - 5x + 6 = 0$ 2, 3
2. $w^2 + 6w + 9 = 0$ -3
3. $b^2 - 3b + 4 = 0$ $\varnothing$

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

4. $p^2 + 4p = 3$ $-5 < p < -4$, $0 < p < 1$
5. $2m^2 + 5 = 10m$ $0 < m < 1$, $4 < m < 5$
6. $2v^2 + 8v = -7$ $-3 < v < -2$, $-2 < v < -1$

NUMBER THEORY For Exercises 7 and 8, use the following information.

Two numbers have a sum of 2 and a product of -8. The quadratic equation $-n^2 + 2n + 8 = 0$ can be used to determine the two numbers.

7. Graph the related function $f(n) = -n^2 + 2n + 8$ and determine its x-intercepts. $-2, 4$
8. What are the two numbers? -2 and 4

DESIGN For Exercises 9 and 10, use the following information.

A footbridge is suspended from a parabolic support. The function $h(x) = -\frac{1}{25}x^2 + 9$ represents the height in feet of the support above the walkway, where $x = 0$ represents the midpoint of the bridge.

9. Graph the function and determine its x-intercepts. $-15, 15$
10. What is the length of the walkway between the two supports? 30 ft

Word Problem Practice
CRM p. 17 OL AL

9-2 Word Problem Practice

Solving Quadratic Equations by Graphing

1. FARMING In order for Ray to decide how much fertilizer to apply to his corn crop this year, he reviews records from previous years. His crop yield y depends on the amount of fertilizer he applies to his fields x according to the equation $y = -x^2 + 4x + 12$. Graph the function, and find the point at which Ray gets the highest yield possible.

2. LIGHT Ayzha and Jeremy hold a flashlight so that the light falls on a piece of graph paper in the shape of a parabola. Ayzha and Jeremy sketch the shape of the parabola and find that the equation $y = x^2 - 3x - 10$ matches the shape of the light beam. Determine the roots of the function. -2 and 5

3. FRAMING A rectangular photograph is 7 inches long and 6 inches wide. The photograph is framed using material x inches wide. If the area of the frame and photograph combined is 156 square inches, what is the width of the framing material? 3 in.

4. WRAPPING PAPER Can a rectangular piece of wrapping paper with an area of 81 square inches have a perimeter of 60 inches? (Hint: Let length $= 30 - w$.) Explain. Solving the equation $(30 - w)w = 81$ gives $w = 3$ or 27. A 3 in. by 27 in. sheet of paper would work.

ENGINEERING For Exercises 5–7, use the following information.

The shape of a satellite dish is often parabolic because of the reflective qualities of parabolas. Suppose a particular satellite dish is modeled by the following equation.

$0.5x^2 = 2 + y$

5. Approximate the solution by graphing. -2 and 2
6. On the coordinate plane above, translate the parabola so that there is a double root. Label this curve A. See students' work.
7. Translate the parabola so that there are no roots. Label this curve B. See students' work.

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

22. $a^2 - 12 = 0$ **23.** $-n^2 + 7 = 0$ **24.** $2c^2 + 20c + 32 = 0$

25. $3s^2 + 9s - 12 = 0$ **26.** $0 = x^2 + 6x + 6$ **27.** $0 = -y^2 + 4y - 1$

28. $-a^2 + 8a = -4$ **29.** $x^2 + 6x = -7$ **30.** $m^2 - 10m = -21$

22–30. See Ch. 9 Answer Appendix.

31. NUMBER THEORY Use a quadratic equation to find two numbers whose sum is 9 and whose product is 20. 4, 5

Cross-Curricular Project

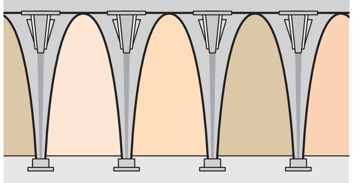
Math Online The graph of the surface areas of the planets can be modeled by a quadratic equation. Visit ca.algebra1.com to continue work on your project.

32. COMPUTER GAMES In a computer football game, the function $-0.005d^2 + 0.22d = h$ simulates the path of a football at the kickoff. In this equation, h is the height of the ball and d is the horizontal distance in yards. What is the horizontal distance the ball will travel before it hits the ground? 44 yd

33. HIKING While hiking in the San Bernardino Mountains, Monya and Kishi stop for lunch on a ledge 1000 feet above a valley. Kishi decides to climb to another ledge 20 feet above Monya. Monya throws an apple up to Kishi, but Kishi misses it. The equation $h = -16t^2 + 30t + 1000$ represents the height in feet above the valley of the apple t seconds after it was thrown. How long did it take for the apple to reach the ground? about 9 s

THEATER For Exercises 34–37, use the following information.

The drama club is building a backdrop using arches whose shape can be represented by the function $f(x) = -x^2 + 2x + 8$, where x is the length in feet. The area under each arch is to be covered with fabric.



34. Graph the quadratic function and determine its x-intercepts. See margin.

35. What is the length of the segment along the floor of each arch? 6 ft

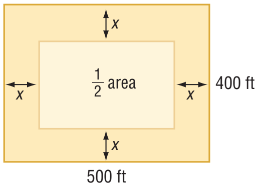
36. What is the height of the arch? 9 ft

37. The formula $A = \frac{2}{3}bh$ can be used to estimate the area A under a parabola. In this formula, b represents the length of the base, and h represents the height. If there are five arches, calculate the total amount of fabric that is needed. about 180 ft^2

39. $(500 - 2x) \cdot (400 - 2x) = 100{,}000$ or $4x^2 - 1800x + 100{,}000 = 0$; about 65 ft

WORK For Exercises 38–40, use the following information.

Kirk and Montega mow the soccer playing fields. They must mow an area 500 feet long and 400 feet wide. They agree that each will mow half the area. Kirk will mow around the edge in a path of equal width until half the area is left.



EXTRA PRACTICE See pages 735, 752.

Math Online Self-Check Quiz at ca.algebra1.com

38. What is the area each person will mow? 100,000 ft^2

39. Write a quadratic equation that could be used to find the width x that Kirk should mow. What width should Kirk mow?

40. The mower can mow a path 5 feet wide. To the nearest whole number, how many times should Kirk go around the field? 13 times

H.O.T. Problems

42. -3, 0, 1; These are the x-intercepts of the graph.

41. OPEN ENDED Draw a graph to show a counterexample to the following statement. Explain. *All quadratic equations have two different solutions.* See Ch. 9 Answer Appendix.

42. CHALLENGE Describe the zeros of $f(x) = \dfrac{x^3 + 2x^2 - 3x}{x + 5}$. Explain your reasoning.

Enrichment
CRM p. 18 OL AL

9-2 Enrichment

Rational Exponents

You have developed the following properties of powers when a is a positive real number and m and n are integers.

$a^m \cdot a^n = a^{m+n}$ $(ab)^m = a^m b^m$ $a^0 = 1$

$(a^m)^n = a^{mn}$ $\frac{a^m}{a^n} = a^{m-n}$ $a^{-m} = \frac{1}{a^m}$

Exponents need not be restricted to integers. We can define rational exponents so that operations involving them will be governed by the properties for integer exponents.

$\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2} \cdot 2} = a$ $\left(a^{\frac{1}{3}}\right)^3 = a^{\frac{1}{3} \cdot 3} = a$ $\left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n} \cdot n} = a$

$a^{\frac{1}{2}}$ squared is a. $a^{\frac{1}{3}}$ cubed is a. $a^{\frac{1}{n}}$ to the n power is a.

$a^{\frac{1}{2}}$ is a square root of a. $a^{\frac{1}{3}}$ is a cube root of a. $a^{\frac{1}{n}}$ is an nth root of a.

$a^{\frac{1}{2}} = \sqrt{a}$ $a^{\frac{1}{3}} = \sqrt[3]{a}$ $a^{\frac{1}{n}} = \sqrt[n]{a}$

BL = Below Grade Level

OL = On Grade Level

AL = Above Grade Level

ELL = English Language Learner

Additional pages not shown:

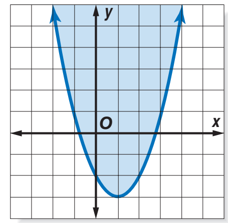
CRM *Lesson Reading Guide*, p. 12 BL OL ELL

CRM *Skills Practice*, p. 15 BL OL

Study Tip

Look Back
To review **linear inequalities,** see Lesson 6–6.

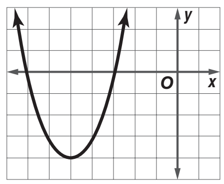
43. CHALLENGE The graph shown is a *quadratic inequality.* Similar to a linear inequality, the quadratic equation is a boundary between two half-planes. Analyze the graph and determine whether the inequality is *always, sometimes,* or *never* greater than 2. Explain. **See margin.**



44. *Writing in Math* Use the information about computer games on page 480 to explain how quadratic equations can be used in computer simulations. Describe what the roots of a simulation equation for a computer golf game represent. **See margin.**

STANDARDS PRACTICE 21.0, 6AF1.1

45. The graph of the equation $y = x^2 + 10x + 21$ is shown. For what value or values of x is $y = 0$? **D**



A $x = -4$ **C** $x = 7$ and $x = 3$

B $x = -5$ **D** $x = -7$ and $x = -3$

46. REVIEW Q-Mart has 1200 blue towels in stock. If they sell half of their towels every three months and do not receive any more shipments of towels, how many towels will they have left after a year? **G**

F 60 **H** 150

G 75 **J** 300

Spiral Review

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each equation. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 9-1) **47–49. See Ch. 9 Answer Appendix.**

47. $y = x^2 + 6x + 9$ **48.** $y = -x^2 + 4x - 3$ **49.** $y = 0.5x^2 - 6x + 5$

Solve each equation. Check your solutions. (Lesson 8-6)

50. $m^2 - 24m = -144$ **{12}** **51.** $7r^2 = 70r - 175$ **{5}** **52.** $4d^2 + 9 = -12d$ $\left\{-\frac{3}{2}\right\}$

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

53. $\frac{10m^4}{30m}$ $\frac{m^3}{3}$ **54.** $\frac{22a^2b^5c^7}{-11abc^2}$ $-2ab^4c^5$ **55.** $\frac{-9m^3n^5}{27m^{-2}n^5y^{-4}}$ $-\frac{m^5y^4}{3}$

★ **56. SHIPPING** An empty book crate weighs 30 pounds. The weight of a book is 1.5 pounds. For shipping, the crate must weigh at least 55 pounds and no more than 60 pounds. What is the acceptable number of books that can be packed in the crate? (Lesson 6-4) **17 to 20 books**

GET READY for the Next Lesson

PREREQUISITE SKILL **Determine whether each trinomial is a perfect square trinomial. If so, factor it.** (Lesson 8-6)

57. $a^2 + 14 + 49$ **yes;** $(a + 7)^2$ **58.** $m^2 - 10m + 25$ **yes;** $(m - 5)^2$ **59.** $t^2 + 16t - 64$ **no** **60.** $4y^2 + 12y + 9$ **yes;** $(2y + 3)^2$

4 Assess

Yesterday's News Ask students to write two ways in which learning to graph quadratic functions in the previous lesson helped them to solve quadratic equations in this lesson.

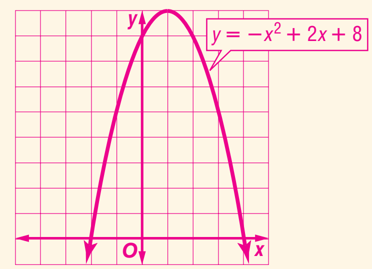
Formative Assessment

Check for student understanding of concepts in Lessons 9-1 and 9-2.

CRM Quiz 1, p. 51

Additional Answers

34.



−2, 4

43. Always; the shaded region of the graph includes y-values greater than 2.

44. Sample answer: Since quadratic functions can be used to model a golf ball after it is hit, solving the related quadratic equation will determine where the ball hits the ground. In the golf problem, one intercept represents the ball's original location and the other intercept represents where the ball hits the ground.

Pre-AP Activity

Use after Example 5

Tell students that in a computer golf game, the function $y = -0.002x^2 + 0.22x$ models the path of a golf ball, where y is the height of the ball and x is the horizontal distance in yards. The green lies uphill from the tee, 90 yards away, atop a hill that has a steady incline of 1 yard per 10 yards of distance. Ask, "Will the ball reach the green without hitting the ground first? Explain your answer." No. On level ground, this ball would travel over 90 yards, but on the uphill slope it will hit the ground after approximately 70 yards.

9-3 Lesson Notes

1 Focus

Standards Alignment

Before Lesson 9-3
Apply basic factoring techniques to second- and simple third-degree polynomials from Standard 1A11.0

Lesson 9-3
Solve a quadratic equation by completing the square from Standard 14.0

After Lesson 9-3
Solve and graph quadratic equations and quadratic equations in the complex number system from Standard 2A8.0

2 Teach

Scaffolding Questions

Have students read *Get Ready for the Lesson*.

Ask:

- Look at the completed square. What is the side length of the square? Remember, each of the smaller squares is made up of 4 unit squares.
  side length $= x + 4$
- What is the area of the square?
  $(x + 4)^2$ or $x^2 + 8x + 16$ units^2

(continued on the next page)

9-3 Solving Quadratic Equations by Completing the Square

Main Ideas

- Solve quadratic equations by finding the square root.
- Solve quadratic equations by completing the square.

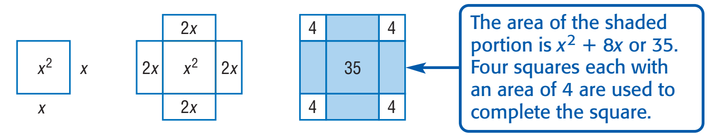
Standard 14.0 **Students solve a quadratic equation** by factoring or **completing the square.** (Key)

New Vocabulary

completing the square

GET READY for the Lesson

Al-Khwarizmi, born in Baghdad in 780, is considered to be one of the foremost mathematicians of all time. He wrote algebra in sentences instead of using equations, and he explained the work with geometric sketches. Al-Khwarizmi would have described $x^2 + 8x = 35$ as "A square and 8 roots are equal to 35 units." He would solve the problem using the following sketch.



To solve problems this way today, you might use algebra tiles or a method called *completing the square*.

Find the Square Root Some equations can be solved by taking the square root of each side.

EXAMPLE Irrational Roots

1 **Solve $x^2 - 10x + 25 = 7$ by taking the square root of each side. Round to the nearest tenth if necessary.**

$x^2 - 10x + 25 = 7$	Original equation
$(x - 5)^2 = 7$	$x^2 - 10x + 25$ is a perfect square trinomial.
$\sqrt{(x - 5)^2} = \sqrt{7}$	Take the square root of each side.
$\lvert x - 5\rvert = \sqrt{7}$	Simplify.
$x - 5 = \pm\sqrt{7}$	Definition of absolute value
$x - 5 + 5 = \pm\sqrt{7} + 5$	Add 5 to each side.
$x = 5 \pm \sqrt{7}$	Simplify.

Use a calculator to evaluate each value of x.

$x = 5 + \sqrt{7}$	or	$x = 5 - \sqrt{7}$	Write each solution.
≈ 7.6		≈ 2.4	Simplify.

The solution set is {2.4, 7.6}.

CHECK Your Progress

1. Solve $m^2 + 18m + 81 = 90$ by taking the square root of each side. Round to the nearest tenth if necessary. **−18.5, 0.5**

Lesson 9-3 Resources

Chapter 9 Resource Masters

Lesson Reading Guide, p. 19 BL OL ELL
Study Guide and Intervention, pp. 20–21 BL OL ELL
Skills Practice, p. 22 BL OL
Practice, p. 23 OL AL
Word Problem Practice, p. 24 OL AL
Enrichment, p. 25 OL AL
Quiz 2, p. 51

Transparencies

5-Minute Check Transparency 9-3

Additional Print Resources

Noteables™ Interactive Study Notebook with Foldables™
Teaching Math With Manipulatives

Technology

ca.algebra1.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

Review Vocabulary

Perfect Square Trinomial a trinomial that is the square of a binomial; *Example:* $x^2 + 12x + 36$ is a perfect square trinomial because it is the square of $(x + 6)$. (Lesson 8-6)

Complete the Square In Example 1, the quadratic expression on one side of the equation was a perfect square. However, few quadratic expressions are perfect squares. To make any quadratic expression a perfect square, a method called **completing the square** may be used.

Consider the pattern for squaring a binomial such as $x + 6$.

$$(x + 6)^2 = x^2 + 2(6)(x) + 6^2$$
$$= x^2 + 12x + 36$$
$$\left(\frac{12}{2}\right)^2 \rightarrow 6^2$$

Notice that one half of 12 is 6 and 6^2 is 36.

KEY CONCEPT *Completing the Square*

To complete the square for a quadratic expression of the form $x^2 + bx$, you can follow the steps below.

Step 1 Find $\frac{1}{2}$ of b, the coefficient of x.

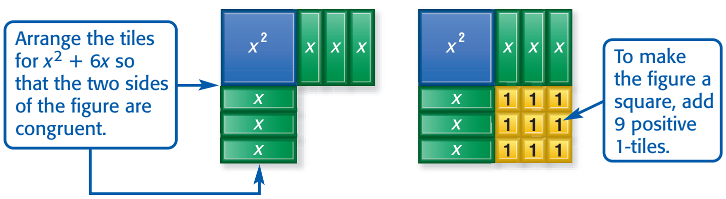
Step 2 Square the result of Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$, the original expression.

EXAMPLE **Complete the Square**

2 **Find the value of c that makes $x^2 + 6x + c$ a perfect square.**

Method 1 Use algebra tiles.



$x^2 + 6x + 9$ is a perfect square.

Method 2 Complete the square.

Step 1	Find $\frac{1}{2}$ of 6.	$\frac{6}{2} = 3$
Step 2	Square the result of Step 1.	$3^2 = 9$
Step 3	Add the result of Step 2 to $x^2 + 6x$.	$x^2 + 6x + 9$

Thus, $c = 9$. Notice that $x^2 + 6x + 9 = (x + 3)^2$.

CHECK Your Progress

2. Find the value of c that makes $r^2 + 8r + c$ a perfect square. **16**

You can use the technique of completing the square to solve quadratic equations.

Concepts in Motion
Animation ca.algebra1.com

ADDITIONAL EXAMPLE

2 Find the value of c that makes $x^2 - 12x + c$ a perfect square. 36

- Graphically, 16 unit squares were added to the original figure to complete the square. How would you change the original equation to represent this addition?
  $x^2 + 8x + 16 = 35 + 16$

Find the Square Root

Example 1 shows how to solve a quadratic equation by taking the square root of each side of an equation.

Formative Assessment

Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLE

1 Solve $x^2 + 6x + 9 = 5$ by taking the square root of each side. Round to the nearest tenth if necessary. $\{-5.2, -0.8\}$

Additional Examples are also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Preventing Errors

Tips for New Teachers

Tell students that when taking the square root of a number, there are two square roots, one positive and one negative. Explain that this is why the plus or minus sign is placed in front of the square root of 7 in Example 1 when the absolute value signs are removed. Without the plus or minus sign, there would be only one solution.

Complete the Square

Example 2 shows how to make any quadratic expression a perfect square by completing the square. **Example 3** shows how to solve a quadratic equation by completing the square. **Example 4** shows how to solve a quadratic equation in which a does not equal one.

ADDITIONAL EXAMPLES

3 Solve $x^2 - 18x + 5 = -12$ by completing the square. $\{1, 17\}$

4 **CANOEING** Suppose the rate of flow of an 80-foot-wide river is given by the equation $r = -0.01x^2 + 0.8x$, where r is the rate in miles per hour and x is the distance from the shore in feet. Joacquim does not want to paddle his canoe against a current that is faster than 5 miles per hour. At what distance from the river bank must he paddle in order to avoid a current of 5 miles per hour? about 7 ft from either bank **Note:** The solutions of the equation are about 7 ft and about 73 ft. Since the river is 80 ft wide, $80 - 73 = 7$.

Differentiated Instruction

Kinesthetic Learners Some students may benefit from using algebra tiles to complete the square when solving quadratic equations like the one in Example 3. Have students use an equation mat. Remind them to add or remove the same number of tiles to each side of the mat.

EXAMPLE Solve an Equation by Completing the Square

3 **Solve $a^2 - 14a + 3 = -10$ by completing the square.**

Isolate the a^2 and a terms. Then complete the square and solve.

$a^2 - 14a + 3 = -10$	Original equation
$a^2 - 14a + 3 - 3 = -10 - 3$	Subtract 3 from each side.
$a^2 - 14a = -13$	Simplify.
$a^2 - 14a + 49 = -13 + 49$	Since $\left(\frac{-14}{2}\right)^2 = 49$, add 49 to each side.
$(a - 7)^2 = 36$	Factor $a^2 - 14a + 49$.
$a - 7 = \pm 6$	Take the square root of each side.
$a = 7 \pm 6$	Add 7 to each side.
$a = 7 + 6$ or $a = 7 - 6$	Separate the solutions.
$= 13$ $\quad = 1$	Simplify.

The solution set is $\{1, 13\}$.

CHECK Your Progress

3. Solve $x^2 - 8x = 4$ by completing the square. Round to the nearest tenth if necessary. **-0.5, 8.5**

To solve a quadratic equation in which the leading coefficient is not 1, first divide each term by the coefficient. Then complete the square.

Real-World Link

One of the exploded fireworks for the Lake Toya Festival in Japan on July 15, 1988, broke a world record. The diameter of the burst was 3937 feet.

Source: *The Guinness Book of Records*

Real-World EXAMPLE Solve a Quadratic Equation in Which $a \neq 1$

4 **ENTERTAINMENT** **The path of debris from fireworks when the wind is about 15 miles per hour can be modeled by the quadratic function $h = -0.04x^2 + 2x + 8$, where h is the height and x is the horizontal distance in feet. How far away from the launch site will the debris land?**

Explore You know the function that relates the horizontal and vertical distances. You want to know how far away the debris will land.

Plan The debris will hit the ground when $h = 0$. Complete the square to solve $-0.04x^2 + 2x + 8 = 0$.

Solve

$-0.04x^2 + 2x + 8 = 0$	Equation for where debris will land
$\frac{-0.04x^2 + 2x + 8}{-0.04} = \frac{0}{-0.04}$	Divide each side by -0.04.
$x^2 - 50x - 200 = 0$	Simplify.
$x^2 - 50x - 200 + 200 = 0 + 200$	Add 200 to each side.
$x^2 - 50x = 200$	Simplify.
$x^2 - 50x + 625 = 200 + 625$	Since $\left(\frac{50}{2}\right)^2 = 625$, add 625 to each side.
$x^2 - 50x + 625 = 825$	Simplify.
$(x - 25)^2 = 825$	Factor $x^2 - 50x + 625$.
$x - 25 = \pm\sqrt{825}$	Take the square root of each side.
$x = 25 \pm \sqrt{825}$	Add 25 to each side.

Pre-AP Activity Use as an Extension

Have students solve $\frac{1}{3}x^2 - \frac{7}{6}x + \frac{1}{2} = 0$ by completing the square. Ask them how this strategy compares to factoring and graphing. $x = \frac{1}{2}$, 3; the equation can be solved more easily by factoring. Graphing may not produce an exact answer if fractions are converted to decimals.

488 Chapter 9 Quadratic and Exponential Functions

Use a calculator to evaluate each value of x.

$x = 25 + \sqrt{825}$ or $x = 25 - \sqrt{825}$ Separate the solutions.

≈ 53.7 ≈ -3.7 Evaluate.

Check Since you are looking for a distance, the negative number is not reasonable. The debris will land about 53.7 feet from the launch site.

CHECK Your Progress

4. Solve $3n^2 - 18n = 30$ by completing the square. Round to the nearest tenth if necessary. **−1.4, 7.4**

Online Personal Tutor at ca.algebra1.com

★ indicates multi-step problem

CHECK Your Understanding

Example 1 (p. 486) **Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.**

1. $b^2 - 6b + 9 = 25$ **−2, 8**
2. $m^2 + 14m + 49 = 20$ **−11.5, −2.5**

Example 2 (p. 487) **Find the value of c that makes each trinomial a perfect square.**

3. $a^2 - 12a + c$ **36**
4. $t^2 + 5t + c$ $\frac{25}{4}$

Example 3 (p. 488) **Solve each equation by completing the square. Round to the nearest tenth if necessary.** **7. −15, 1**

5. $c^2 - 6c = 7$ **−1, 7**
6. $x^2 + 7x = -12$ **−4, −3**
7. $v^2 + 14v - 9 = 6$
8. $r^2 - 4r = 2$ **−0.4, 4.4**
9. $4a^2 + 9a - 1 = 0$ **−2.4, 0.1**
10. $7 = 2p^2 - 5p + 8$ **0.2, 2.3**

Example 4 (pp. 488–489) ★ 11. **GEOMETRY** The area of a square can be doubled by increasing the length by 6 inches and the width by 4 inches. What is the length of the side of the square? **12 in.**

Exercises

HOMEWORK HELP

For Exercises	See Examples
12–15	1
16–19	2
20–27	3
28–33	4

Exercise Levels
A: 12–33
B: 34–42
C: 43–47

23. −27, 7
28. −1.4, 3.4

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

12. $b^2 - 4b + 4 = 16$ **−2, 6**
13. $t^2 + 2t + 1 = 25$ **−6, 4**
14. $g^2 - 8g + 16 = 2$ **2.6, 5.4**
15. $w^2 + 16w + 64 = 18$ **−12.2, −3.8**

Find the value of c that makes each trinomial a perfect square.

16. $s^2 - 16s + c$ **64**
17. $y^2 - 10y + c$ **25**
18. $p^2 - 7p + c$ $\frac{49}{4}$
19. $c + 11k + k^2$ $\frac{121}{4}$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

20. $s^2 - 4s - 12 = 0$ **−2, 6**
21. $d^2 + 3d - 10 = 0$ **−5, 2**
22. $y^2 - 19y + 4 = 70$ **−3, 22**
23. $d^2 + 20d + 11 = 200$
24. $a^2 - 5a = -4$ **1, 4**
25. $p^2 - 4p = 21$ **−3, 7**
26. $x^2 + 4x + 3 = 0$ **−3, −1**
27. $d^2 - 8d + 7 = 0$ **1, 7**
28. $5s^2 - 10s = 23$
29. $9r^2 + 49 = 42r$ **2.3**
30. $4h^2 + 25 = 20h$ **2.5**
31. $9w^2 - 12w - 1 = 0$ **−0.1, 1.4**

Focus on Mathematical Content

Irrational Solutions Completing the square to solve a quadratic equation does not mean that the solutions will be integers. If the equation already has a constant term, it is likely that after completing the square, the constant will not be a perfect square, and the solutions will be irrational.

Tips for New Teachers — Student Misconceptions

Tell students not to throw out negative solutions to real-world problems. Remind them that they must first examine the problem to see if the solution fits the situation.

3 Practice

Formative Assessment

Use Exercises 1–11 to check for understanding.

Odd/Even Assignments

Exercises 12–33 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Tips for New Teachers — Preventing Errors

For Exercises 20–31, remind students that the amount that they add to one side of the equation to complete the square must also be added to the other side of the equation.

DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	12–33, 43, 44, 46–65	13–33 odd, 48, 49	12–32 even, 43, 44, 46, 47, 50–65
OL Core	13–41 odd, 42–44, 46–65	12–33, 48, 49	34–44, 46, 47, 50–65
AL Advanced /Pre-AP	34–61 (optional: 62–65)		

Study Guide and Intervention

CRM pp. 20–21 OL AL ELL

9-3 Study Guide and Intervention

Solving Quadratic Equations by Completing the Square

Find the Square Root An equation such as $x^2 - 4x + 4 = 5$ can be solved by taking the square root of each side.

Example 1 **Solve $x^2 - 2x + 1 = 9$. Round to the nearest tenth if necessary.**

$$x^2 - 2x + 1 = 9$$
$$(x - 1)^2 = 9$$
$$\sqrt{(x - 1)^2} = \sqrt{9}$$
$$|x - 1| = \sqrt{9}$$
$$x - 1 = \pm 3$$
$$x - 1 + 1 = \pm 3 + 1$$
$$x = 1 \pm 3$$

$x = 1 + 3$ or $x = 1 - 3$
$= 4$ $\quad= -2$

The solution set is $\{-2, 4\}$.

Example 2 **Solve $x^2 - 4x + 4 = 5$. Round to the nearest tenth if necessary.**

$$x^2 - 4x + 4 = 5$$
$$(x - 2)^2 = 5$$
$$\sqrt{(x - 2)^2} = \sqrt{5}$$
$$|x - 2| = \sqrt{5}$$
$$x - 2 = \pm\sqrt{5}$$
$$x - 2 + 2 = \pm\sqrt{5} + 2$$
$$x = 2 \pm \sqrt{5}$$

Use a calculator to evaluate each value of x.

$x = 2 + \sqrt{5}$ or $x = 2 - \sqrt{5}$
≈ 4.2 $\quad\approx -0.2$

The solution set is $\{-0.2, 4.2\}$.

Exercises

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $x^2 + 4x + 4 = 9$ **−5, 1**
2. $m^2 + 12m + 36 = 1$ **−7, −5**
3. $r^2 - 6r + 9 = 16$ **−1, 7**
4. $x^2 - 2x + 1 = 25$ **−4, 6**
5. $x^2 - 8x + 16 = 5$ **1.8, 6.2**
6. $x^2 - 10x + 25 = 8$ **2.2, 7.8**
7. $c^2 - 4c + 4 = 7$ **−0.6, 4.6**
8. $p^2 + 16p + 64 = 3$ **−9.7, −6.3**
9. $x^2 + 8x + 16 = 9$ **−7, −1**
10. $x^2 + 6x + 9 = 4$ **−5, −1**
11. $a^2 + 8a + 16 = 10$ **−7.2, −0.8**
12. $y^2 - 12y + 36 = 5$ **3.8, 8.2**
13. $x^2 + 10x + 25 = 1$ **−6, −4**
14. $y^2 + 14y + 49 = 6$ **−9.4, −4.6**
15. $m^2 - 8m + 16 = 2$ **2.6, 5.4**
16. $x^2 + 12x + 36 = 10$ **−9.2, −2.8**
17. $a^2 - 14a + 49 = 3$ **5.3, 8.7**
18. $y^2 + 8y + 16 = 7$ **−6.6, −1.4**

Practice

CRM p. 23 OL AL

9-3 Practice

Solving Quadratic Equations by Completing the Square

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $b^2 - 14b + 49 = 64$ **−1, 15**
2. $s^2 + 16s + 64 = 100$ **−18, 2**
3. $h^2 - 8h + 16 = 15$ **0.1, 7.9**
4. $a^2 + 6a + 9 = 27$ **−8.2, 2.2**
5. $p^2 - 20p + 100 = 28$ **4.7, 15.3**
6. $u^2 + 10u + 25 = 90$ **−14.5, 4.5**

Find the value of c that makes each trinomial a perfect square.

7. $t^2 - 24t + c$ **144**
8. $b^2 + 28b + c$ **196**
9. $y^2 + 40y + c$ **400**
10. $m^2 + 3m + c$ $\mathbf{\frac{9}{4}}$
11. $g^2 - 9g + c$ $\mathbf{\frac{81}{4}}$
12. $v^2 - v + c$ $\mathbf{\frac{1}{4}}$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

13. $w^2 - 14w + 24 = 0$ **2, 12**
14. $p^2 + 12p = 13$ **−13, 1**
15. $s^2 - 30s + 56 = -25$ **3, 27**
16. $v^2 + 8v + 9 = 0$ **−6.6, −1.4**
17. $t^2 - 10t + 6 = -7$ **1.5, 8.5**
18. $n^2 + 18n + 50 = 9$ **−15.3, −2.7**
19. $3u^2 + 15u - 3 = 0$ **−5.2, 0.2**
20. $4c^2 - 72 = 24c$ **−2.2, 8.2**
21. $0.9a^2 + 5.4a - 4 = 0$ $\mathbf{-6\frac{2}{3}, \frac{2}{3}}$
22. $0.4h^2 + 0.8h = 0.2$ **−2.2, 0.2**
23. $\frac{1}{2}x^2 - \frac{1}{2}x - 10 = 0$ **−4, 5**
24. $\frac{1}{4}x^2 + \frac{3}{2}x - 2 = 0$ **−7.1, 1.1**

Lesson 9-3

BUSINESS For Exercises 25 and 26, use the following information.
Jaime owns a business making decorative boxes to store jewelry, mementos, and other valuables. The function $y = x^2 + 50x + 1800$ models the profit y that Jaime has made in month x for the first two years of his business.

25. Write an equation representing the month in which Jaime's profit is $2400. $\mathbf{x^2 + 50x + 1800 = 2400}$
26. Use completing the square to find out in which month Jaime's profit is $2400. **the tenth month**
27. **PHYSICS** From a height of 256 feet above a lake on a cliff, Mikaela throws a rock out over the lake. The height H of the rock t seconds after Mikaela throws it is represented by the equation $H = -16t^2 + 32t + 256$. To the nearest tenth of a second, how long does it take the rock to reach the lake below? (*Hint:* Replace H with 0.) **5.1 s**

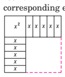
Word Problem Practice

CRM p. 24 OL AL

9-3 Word Problem Practice

Solving Quadratic Equations by Completing the Square

1. **INTERIOR DESIGN** Modular carpeting is installed in small pieces rather than as a large roll so that only a few pieces need to be replaced if a small area is damaged. Suppose the room shown in the diagram below is being fitted with modular carpeting. Complete the square to determine the number of 1 ft by 1 ft squares of carpeting needed to finish the room. Fill in the missing terms in the corresponding equation below. **25; 5**



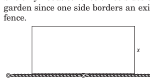
$x^2 + 10x +$ _____ $= (x +$ _____$)^2$

2. **FALLING OBJECTS** Keisha throws a rock down an old well. The distance d (in feet) the rock falls after t seconds can be represented by the equation $d = 16t^2 + 64t$. If the water in the well is 80 feet below ground, how many seconds will it take for the rock to hit the water? **1 second**

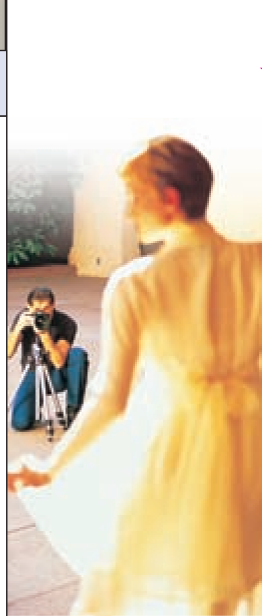
3. **MARS** On Mars, the gravity acting on an object is less than that on Earth. On Earth, a golf ball hit with an initial upward velocity of 26 meters per second will reach a maximum height of about 34.5 meters. The height h of an object on Mars that leaves the ground with an initial velocity of 26 meters per second is given by the equation $h = -1.9t^2 + 26t$. Find the maximum height if the same golf ball is hit on Mars. Round your answer to the nearest tenth. **88.9 m**

4. **FROGS** A frog sitting on a stump 3 feet high hops off and lands on the ground. During its leap, its height h (in feet) is given by $h = -0.5d^2 + 2d + 3$, where d is the distance from the base of the stump. How far is the frog from the base of the stump when it landed on the ground? $\mathbf{2 + \sqrt{10}}$ **or about 5.16 ft**

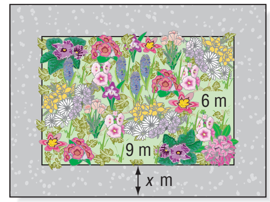
GARDENING For exercises 5–7, use the following information.
Peg is planning a rectangular vegetable garden using 250 feet of fencing material. She only needs to fence three sides of the garden since one side borders an existing fence.



5. Let x = the width of the rectangle. Write an expression to represent the area of the garden if she uses all the fencing material. $\mathbf{x(250 - 2x)}$
6. Find the vertex of the equation and identify it as a maximum or a minimum. **(62.5, 7812.5); maximum**
7. Interpret the vertex of the equation in terms of the situation. **If the short side is 62.5 ft and the other side 125 ft, the garden will be the largest possible, with an area of 7812.5 ft².**



32. ★ **PARK PLANNING** A rectangular garden of wild flowers is 9 meters long by 6 meters wide. A pathway of constant width goes around the garden. If the area of the path equals the area of the garden, what is the width of the path? **1.5 m**



33. **NUTRITION** The consumption of bread and cereal in the United States is increasing and can be modeled by the function $y = 0.059x^2 - 7.423x + 362.1$, where y represents the consumption of bread and cereal in pounds and x represents the number of years since 1900. If this trend continues, in what future year will the average American consume 300 pounds of bread and cereal? **about 2017**

Real-World Career...
Photographer
Photographers must consider lighting, lens setting, and composition to create the best photograph.

Math Online
For more information, go to ca.algebra1.com.

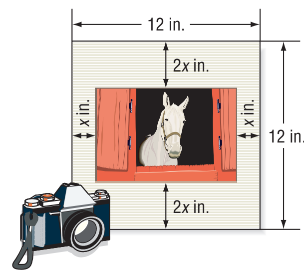
Solve each equation by completing the square. Round to the nearest tenth if necessary.

34. $0.3t^2 + 0.1t = 0.2$ **−1, 0.7**
35. $0.4v^2 + 2.5 = 2v$ **2.5**
36. $\frac{1}{2}d^2 - \frac{5}{4}d - 3 = 0$ $\mathbf{-\frac{3}{2}, 4}$
37. $\frac{1}{3}f^2 - \frac{7}{6}f + \frac{1}{2} = 0$ $\mathbf{\frac{1}{2}, 3}$
38. Find all values of c that make $x^2 + cx + 81$ a perfect square. **−18, 18**
39. Find all values of c that make $x^2 + cx + 144$ a perfect square. **−24, 24**

Solve each equation for x in terms of c by completing the square.

40. $x^2 + 4x + c = 0$ $\mathbf{-2 \pm \sqrt{4 - c}}$
41. $x^2 - 6x + c = 0$ $\mathbf{3 \pm \sqrt{9 - c}}$

42. ★ **PHOTOGRAPHY** Emilio is placing a photograph behind a 12-inch-by-12-inch piece of matting. The photograph is to be positioned so that the matting is twice as wide at the top and bottom as it is at the sides. If the area of the photograph is to be 54 square inches, what are the dimensions? **9 in. by 6 in.**



EXTRA PRACTICE
See pages 735, 752.
Math Online
Self-Check Quiz at ca.algebra1.com

H.O.T. Problems

43. **OPEN ENDED** Make a square using one or more of each of the following types of tiles. **See margin.**
- x^2-tile
- x-tile
- 1-tile

Describe the area of your square using an algebraic expression.

44. **REASONING** Compare and contrast the following strategies for solving $x^2 - 5x - 7 = 0$: completing the square, graphing the related function, and factoring.

44. Sample answer: Completing the square gives an exact solution. Graphing $f(x) = x^2 - 5x - 7$ does not give an exact solution. Since $x^2 - 5x - 7$ cannot be factored, this strategy cannot be used in this case.

45. **CHALLENGE** Without graphing, describe the solution of $x^2 + 4x + 12 = 0$. Explain your reasoning. Then describe the graph of the related function. **See margin.**

46. **Which One Doesn't Belong?** Identify the expression that does not belong with the other three. Explain your reasoning. **See margin.**

$n^2 - n + \frac{1}{4}$	$n^2 + n + \frac{1}{4}$	$n^2 - \frac{2}{3}n + \frac{1}{9}$	$n^2 + \frac{1}{3}n + \frac{1}{9}$

Enrichment

CRM p. 25 OL AL

9-3 Enrichment

Parabolas Through Three Given Points

If you know two points on a straight line, you can find the equation of the line. To find the equation of a parabola, you need three points on the curve.

Here is how to approximate an equation of the parabola through the points (0, −2), (3, 0), and (5, 2).

Use the general equation $y = ax^2 + bx + c$. By substituting the given values for x and y, you get three equations.

(0, −2): $-2 = c$
(3, 0): $0 = 9a + 3b + c$
(5, 2): $2 = 25a + 5b + c$

First, substitute −2 for c in the second and third equations. Then solve those two equations as you would any system of two equations. Multiply the second equation by 5 and the third equation by −3.

BL = Below Grade Level
OL = On Grade Level
AL = Above Grade Level
ELL = English Language Learner

Additional pages not shown:

CRM ***Lesson Reading Guide,*** p. 19 BL OL ELL
CRM ***Skills Practice,*** p. 22 BL OL

47. ***Writing in Math*** Use the information about Al-Khwarizmi on page 486 to explain how ancient mathematicians used squares to solve algebraic equations. Include an explanation of Al-Khwarizmi's drawings for $x^2 + 8x = 35$ and a step-by-step algebraic solution with justification for each step of the equation. **See Ch. 9 Answer Appendix.**

STANDARDS PRACTICE 4.0, 7AF1.2

48. What are the solutions to the quadratic equation $p^2 - 14p = 32$? **C**

A 16 C $-2, 16$

B $-3, 14$ D $-4, 7$

49. **REVIEW** If $a = -5$ and $b = 6$, then $3a - 2ab =$ **J**

F -75 H 30

G -55 J 45

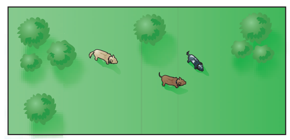
Spiral Review

Solve each equation by graphing. (Lesson 9-2) **50–52. See margin.**

50. $x^2 + 7x + 12 = 0$ **51.** $x^2 - 16 = 0$ **52.** $x^2 - 2x + 6 = 0$

PARKS **For Exercises 53 and 54, use the following information.** (Lesson 9-1)

A city is building a dog park that is rectangular in shape and measures 280 feet around three of the four sides as shown in the diagram.



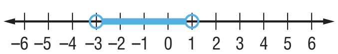
53. If the width of the park in feet is x, write an equation that models the area A of the park. $A = (280 - 2x)x$ **or** $A = 280x - 2x^2$

54. Analyze the graph of the related function by finding the coordinates of the vertex and describing what this point represents. **The vertex is at (70, 9800). This means that the width of the park with the greatest area is 70 feet and the area is 9800 square feet.**

Find the GCF for each set of monomials. (Lesson 8-1)

55. $14a^2b^3, 20a^3b^2c, 35ab^3c^2$ ab^2 **56.** $32m^2n^3, 8m^2n, 56m^3n^2$ $8m^2n$

Write an inequality for each graph. (Lesson 6-4)

57.


$-3 < x < 1$

58.


$x \le -2$ **or** $x > 1$

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 5-2)

59. $y = 2x$ **(3, 6)**
$x + y = 9$

60. $x = y + 3$ **(4, 1)**
$2x - 3y = 5$

61. $x - 2y = 3$ **(7, 2)**
$3x + y = 23$

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate $\sqrt{b^2 - 4ac}$ for each set of values. Round to the nearest tenth if necessary.** (Lesson 1–2)

62. $a = 1, b = -2, c = -15$ **8** **63.** $a = 2, b = 7, c = 3$ **5**

64. $a = 1, b = 5, c = -2$ **5.7** **65.** $a = -2, b = 7, c = 5$ **9.4**

4 Assess

Name the Math Ask students what mathematical procedures they would use to solve a quadratic equation by completing the squares.

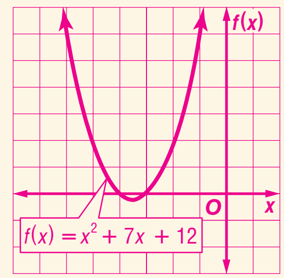
Formative Assessment

Check for student understanding of concepts in Lesson 9-3.

CRM Quiz 2, p. 51

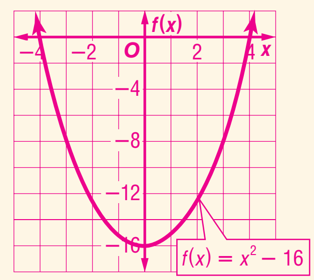
Additional Answers

50.



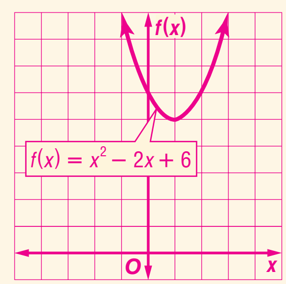
$-4, -3$

51.



$-4, 4$

52.



$\varnothing$

43. Sample answer: $x^2 + 4x + 4$

x	1	1
x	1	1
x^2	x	x

45. There are no real solutions since completing the square results in $(x + 2)^2 = -8$ and the square of a number cannot be negative.

46. $n^2 + \frac{1}{3}n + \frac{1}{9}$; It is the only trinomial that is not a perfect square.

CHAPTER 9 Mid-Chapter Quiz

Formative Assessment

Use the Mid-Chapter Quiz to assess students' progress in the first half of the chapter.

For problems answered incorrectly, have students review the lessons indicated in parentheses.

Summative Assessment

CRM Mid-Chapter Test, p. 53

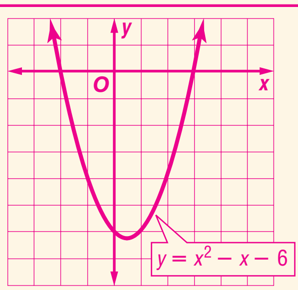
ExamView Assessment Suite Customize and create multiple versions of your Mid-Chapter Tests and their answer keys.

FOLDABLES Study Organizer Foldables™ Follow-Up

Before students complete the Mid-Chapter Quiz, encourage them to review the information on the pages labeled 9-1 through 9-3 of their Foldables.

Additional Answers

1.



$x = 0.5$; $(0.5, -6.25)$; minimum

For Exercises 2–3, 5–7, see Ch. 9 Answer Appendix.

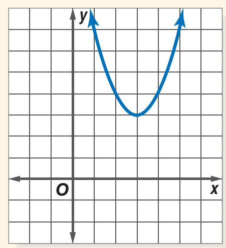
CHAPTER 9 Mid-Chapter Quiz

Lessons 9-1 through 9-3

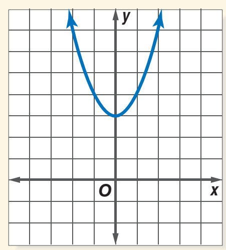
Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 9-1) **1–3. See margin.**

1. $y = x^2 - x - 6$
2. $y = 2x^2 + 3$
3. $y = -3x^2 - 6x + 5$
4. **MULTIPLE CHOICE** Which graph shows a function $y = x^2 + b$ when $b > 1$? (Lesson 9-1) **B**

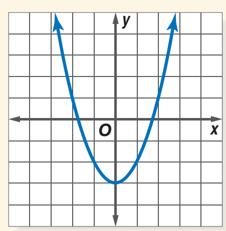
A



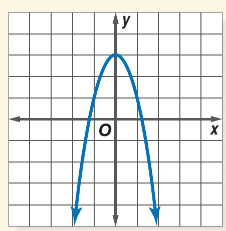
B



C



D



Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie. (Lesson 9-2) **5–7. See margin.**

5. $x^2 + 6x + 10 = 0$
6. $x^2 - 2x - 1 = 0$
7. $x^2 - 5x - 6 = 0$

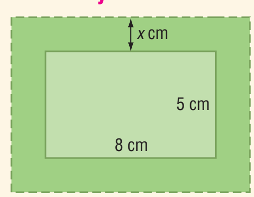
8. **SOFTBALL** In a softball game, Lola hit the ball straight up with an initial upward velocity of 47 feet per second. The height h of the softball in feet above ground after t seconds can be modeled by the equation $h = -16t^2 + 47t + 3$. How long was the softball in the air before it hit the ground? (Lesson 9-2) **3 s**

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 9-3)

9. $s^2 + 8s = -15$ **−5, −3**
10. $a^2 - 10a = -24$ **4, 6**
11. $y^2 - 14y + 49 = 5$ **4.8, 9.2**
12. $2b^2 - b - 7 = 14$ **−3, 3.5**

13. **ROCKETS** A model rocket is launched from the ground with an initial upward velocity of 475 feet per second. About how many seconds will it take to reach the ground? Use the formula $h = -16t^2 + 175t$, where h is the height of the rocket and t is the time in seconds. Round to the nearest tenth if necessary. (Lesson 9-3) **10.9 s**

14. **GEOMETRY** The length and width of the rectangle are increased by the same amount so that the new area is 154 square centimeters. Find the dimensions of the new rectangle. (Lesson 9-3) **11 cm by 14 cm**



Data-Driven Decision Making

Diagnostic Teaching Based on the results of the Chapter 9 Mid-Chapter Quiz, use the following to review concepts that students continue to find challenging.

Exercises	Lesson	Standard	Resources for Review
1–4	9–1	21.0	CRM Study Guide and Intervention pp. 6–7, 13–14, 20–21 Math Online • Extra Examples • Personal Tutor • Concepts in Motion
5–8	9–2	21.0	
9–14	9–3	14.0	

492 Chapter 9 Quadratic and Exponential Functions

9-4 Solving Quadratic Equations by Using the Quadratic Formula

Main Ideas

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number of solutions for a quadratic equation.

Standard 19.0 **Students know the quadratic formula** and are familiar with its proof by completing the square. (Key)

Standard 20.0 **Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.** (Key)

Standard 22.0 **Students use the quadratic formula** or factoring techniques or both **to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.**

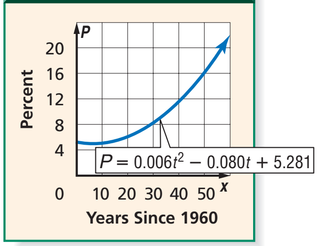
New Vocabulary

Quadratic Formula
discriminant

GET READY for the Lesson

In the past few decades, there has been a dramatic increase in the percent of people living in the United States who were born in other countries. This trend can be modeled by the quadratic function $P = 0.006t^2 - 0.080t + 5.281$, where P is the percent born outside the United States and t is the number of years since 1960.

Percent Born Outside the U.S.



To predict when 15% of the population will be people who were born outside of the U.S., you can solve the equation $15 = 0.006t^2 - 0.080t + 5.281$. This equation would be impossible or difficult to solve using factoring, graphing, or completing the square.

Quadratic Formula You can solve the standard form of the quadratic equation $ax^2 + bx + c = 0$ for x. The result is the **Quadratic Formula**.

KEY CONCEPT — *The Quadratic Formula*

The solutions of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You can solve quadratic equations by factoring, graphing, completing the square, or using the Quadratic Formula.

Study Tip

Quadratic Formula

The Quadratic Formula is proved in Lesson 10–1.

EXAMPLE Solve Quadratic Equations

1 **Solve each equation. Round to the nearest tenth if necessary.**

a. $x^2 - 2x - 24 = 0$

Method 1 Factoring

$x^2 - 2x - 24 = 0$ Original equation

$(x + 4)(x - 6) = 0$ Factor $x^2 - 2x - 24$.

$x + 4 = 0$ or $x - 6 = 0$ Zero Product Property

$x = -4$ $\quad x = 6$ Solve for x.

(continued on the next page)

9-4 Lesson Notes

1 Focus

Standards Alignment

Before Lesson 9-4
Use the correct order of operations to evaluate algebraic expressions from Standard 7AF1.2

Lesson 9-4
Know the quadratic formula.
Use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations from Standards 19.0, 20.0, and 22.0

After Lesson 9-4
Solve and graph quadratic equations and quadratic equations in the complex number system from Standard 2A8.0

2 Teach

Scaffolding Questions Have students read *Get Ready for the Lesson*.

Ask:

- Since this population trend is represented by a quadratic equation, what does it assume about the percent of people who were born in other countries in the past? It was higher in the past, then bottomed out and has since been rising.

(continued on the next page)

Lesson 9-4 Resources

Chapter 9 Resource Masters
Lesson Reading Guide, p. 26 BL OL ELL
Study Guide and Intervention, pp. 27–28 BL OL ELL
Skills Practice, p. 29 BL OL
Practice, p. 30 OL AL
Word Problem Practice, p. 31 OL AL
Enrichment, p. 32 OL AL

Transparencies
5-Minute Check Transparency 9-4

Additional Print Resources
Noteables™ Interactive Study Notebook with Foldables™

Technology
ca.algebra1.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

- Why would the equation be difficult to solve using factoring or completing the square? The fractional values of the coefficients would make using the methods very difficult, if not impossible.

Quadratic Formula

Example 1 shows how to use the Quadratic Formula to solve a quadratic equation with integral roots. **Example 2** shows how to use the Quadratic Formula to solve a real-world problem.

Formative Assessment

Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLE

1 Solve each equation. Round to the nearest tenth if necessary.

a. $x^2 - 2x - 35 = 0$ $\{-5, 7\}$

b. $15x^2 - 8x = 4$ $\{-0.3, 0.8\}$

Additional Examples are also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Preventing Errors

Tips for New Teachers

Some students may notice that the trinomial in Example 1 can be factored. Explain that an equation that could be factored was used to demonstrate that the Quadratic Formula produces the correct solutions. Tell students that if they see an easier way (such as factoring) to solve a quadratic equation, they should use the easier method.

Method 2 Quadratic Formula

For this equation, $a = 1$, $b = -2$, and $c = -24$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-24)}}{2(1)} \qquad a = 1, b = -2, \text{ and } c = -24$$

$$= \frac{2 \pm \sqrt{4 + 96}}{2} \qquad \text{Multiply.}$$

$$= \frac{2 \pm \sqrt{100}}{2} \text{ or } \frac{2 \pm 10}{2} \qquad \text{Add and simplify.}$$

$$x = \frac{2 - 10}{2} \text{ or } x = \frac{2 + 10}{2} \qquad \text{Separate the solutions.}$$

$$= -4 \qquad = 6 \qquad \text{Simplify.}$$

The solution set is $\{-4, 6\}$.

Study Tip

The Quadratic Formula

You may want to simplify this equation by dividing each side by 2 before applying the Quadratic Formula. However, the Quadratic Formula can help you find the solution of *any* quadratic equation.

b. $24x^2 - 14x = 6$

Step 1 Rewrite the equation in standard form.

$$24x^2 - 14x = 6 \qquad \text{Original equation}$$

$$24x^2 - 14x - 6 = 0 \qquad \text{Subtract 6 from each side.}$$

Step 2 Apply the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(24)(-6)}}{2(24)} \qquad a = 24, b = -14, \text{ and } c = -6$$

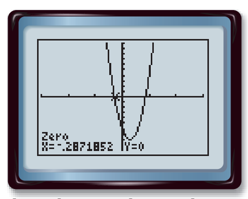
$$= \frac{14 \pm \sqrt{196 + 576}}{48} \qquad \text{Multiply.}$$

$$= \frac{14 \pm \sqrt{772}}{48} \qquad \text{Add.}$$

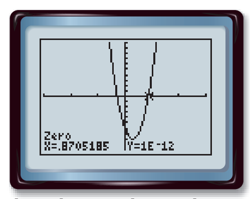
$$x = \frac{14 - \sqrt{772}}{48} \text{ or } x = \frac{14 + \sqrt{772}}{48} \qquad \text{Separate the solutions.}$$

$$\approx -0.3 \qquad \approx 0.9 \qquad \text{Simplify.}$$

You can use a graphing calculator to check the solutions. Use the CALC menu to determine the zeros of the related quadratic function.



[−3, 3] scl: 1 by [−10. 10] scl: 1



[−3, 3] scl: 1 by [−10. 10] scl: 1

To the nearest tenth, the solution set is $\{-0.3, 0.9\}$.

CHECK Your Progress

1A. $x^2 + 3x - 18 = 0$ **−6, 3** **1B.** $4x^2 + 2x = 17$ **−2.3, 1.8**

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Focus on Mathematical Content

The Quadratic Formula Even though the Quadratic Formula may not be the easiest way to solve some quadratic equations, it always works. The derivation of the formula is the solution of the equation $ax^2 + bx + c = 0$ by completing the square. The deviation is shown in Lesson 10-1 when students learn about simplifying radical expressions.

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The table summarizes the five methods for solving quadratic equations.

CONCEPT SUMMARY *Solving Quadratic Equations*

Method	Can Be Used	Comments	Lesson(s)
factoring	sometimes	Use if constant term is 0 or factors are easily determined.	8-2 to 8-6
using a table	sometimes	Not always exact; use only when an approximate solution is sufficient.	9-2
graphing	always	Not always exact; use only when an approximate solution is sufficient.	9-2
completing the square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is an even number.	9-3
Quadratic Formula	always	Other methods may be easier to use in some cases, but this method always gives accurate solutions.	9-4

Real-World Link

Astronauts have found walking on the Moon to be very different from walking on Earth because the gravitational pull of the Moon is only 1.6 meters per second squared. The gravitational pull on Earth is 9.8 meters per second squared.

Source: *World Book Encyclopedia*

Real-World EXAMPLE Use the Quadratic Formula to Solve a Problem

2 **SPACE TRAVEL** **The height H of an object t seconds after it is propelled upward with an initial velocity v is represented by $H = -\frac{1}{2}gt^2 + vt + h$, where g is the gravitational pull and h is the initial height. Suppose an astronaut on the Moon throws a baseball upward with an initial velocity of 10 meters per second, letting go of the ball 2 meters above the ground. Use the information at the left to find how much longer the ball will stay in the air than a similarly thrown baseball on Earth.**

In order to find when the ball hits the ground, you must find when $H = 0$. Write two equations to represent the situation on the Moon and on Earth.

Baseball Thrown on the Moon

$$H = -\frac{1}{2}gt^2 + vt + h$$
$$0 = -\frac{1}{2}(1.6)t^2 + 10t + 2$$
$$0 = -0.8t^2 + 10t + 2$$

Baseball Thrown on Earth

$$H = -\frac{1}{2}gt^2 + vt + h$$
$$0 = -\frac{1}{2}(9.8)t^2 + 10t + 2$$
$$0 = -4.9t^2 + 10t + 2$$

To find accurate solutions, use the Quadratic Formula.

Moon:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-10 \pm \sqrt{10^2 - 4(-0.8)(2)}}{2(-0.8)}$$
$$= \frac{-10 \pm \sqrt{106.4}}{-1.6}$$
$$t \approx 12.7 \quad \text{or} \quad t \approx -0.2$$

Earth:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-10 \pm \sqrt{10^2 - 4(-4.9)(2)}}{2(-4.9)}$$
$$= \frac{-10 \pm \sqrt{139.2}}{-9.8}$$
$$t \approx 2.2 \quad \text{or} \quad t \approx -0.2$$

Since a negative time is not reasonable, use the positive solutions. The ball will stay in the air about 12.7 – 2.2 or 10.5 seconds longer on the Moon.

CHECK Your Progress

2. **GEOMETRY** The perimeter of a rectangle is 60 inches. Find the dimensions of the rectangle if its area is 221 square inches.

13 in. by 17 in.

Intervention

Using the Quadratic Formula

Tell students that it is best to simplify the Quadratic Formula one step at a time. For example, students should first simplify under the radical sign, then find the square root, then simplify the numerator and denominator, and finally perform the division. Skipping steps may introduce errors.

Solving Quadratic Equations

While the chart on this page offers suggestions for when to use each quadratic method, do not hold students to using all methods. Some students do not yet have the mathematical maturity to analyze each equation and determine the best method to use to save time.

ADDITIONAL EXAMPLE

2 **SPACE TRAVEL** Two possible future destinations for astronauts are the planet Mars and Europa, a moon of the planet Jupiter. The gravitational pull on Mars is about 3.7 meters per second squared; on Europa, it is 1.3 meters per second squared. Using the information and equation from Example 3, find how much longer baseballs thrown on Mars and Europa will stay above the ground than baseballs thrown on Earth. The ball thrown on Mars will stay aloft about 3.4 seconds longer than the ball thrown on Earth. The ball thrown on Europa will stay aloft 13.4 seconds longer than the ball thrown on Earth.

The Discriminant

Example 3 shows how to use the discriminant to determine the number of real roots for a quadratic equation.

ADDITIONAL EXAMPLE

3 State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

a. $3x^2 + 10x = 12$ discriminant: 244; two real roots

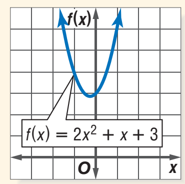
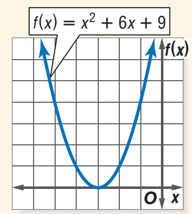
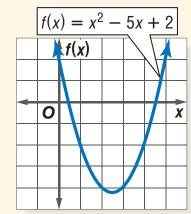
b. $4x^2 - 2x + 14 = 0$ discriminant: –220; no real roots

Tips for New Teachers — Preventing Errors

Remind students to be careful to include any negative signs when finding the discriminant. One missed negative sign can turn the discriminant from positive to negative and yield an incorrect result.

The Discriminant In the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The value of the discriminant can be used to determine the number of real roots for a quadratic equation.

KEY CONCEPT *Using the Discriminant*

Discriminant	negative	zero	positive
Example	$2x^2 + x + 3 = 0$ $x = \frac{-1 \pm \sqrt{1^2 - 4(2)(3)}}{2(2)}$ $= \frac{-1 \pm \sqrt{-23}}{4}$ There are no real roots since no real number can be the square root of a negative number.	$x^2 + 6x + 9 = 0$ $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$ $= \frac{-6 \pm \sqrt{0}}{2}$ $= \frac{-6}{2}$ or -3 There is a double root, -3.	$x^2 - 5x + 2 = 0$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$ $= \frac{5 \pm \sqrt{17}}{2}$ There are two roots, $\frac{5 + \sqrt{17}}{2}$ and $\frac{5 - \sqrt{17}}{2}$.
Graph of Related Function	 The graph does not cross the x-axis.	 The graph touches the x-axis in one place.	 The graph crosses the x-axis twice.
Number of Real Roots	0	1	2

EXAMPLE 3 Use the Discriminant

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

a. $2x^2 + 10x + 11 = 0$

$b^2 - 4ac = 10^2 - 4(2)(11)$ $a = 2$, $b = 10$, and $c = 11$

$= 12$ Simplify.

Since the discriminant is positive, the equation has two real roots.

b. $3m^2 + 4m = -2$

Step 1 Rewrite the equation in standard form.

$3m^2 + 4m = -2$ Original equation

$3m^2 + 4m + 2 = -2 + 2$ Add 2 to each side.

$3m^2 + 4m + 2 = 0$ Simplify.

Step 2 Find the discriminant.

$b^2 - 4ac = 4^2 - 4(3)(2)$ $a = 3$, $b = 4$, and $c = 2$

$= -8$ Simplify.

Since the discriminant is negative, the equation has no real roots.

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CHECK Your Progress 3A. 0; one real root

3A. $4n^2 - 20n + 25 = 0$ **3B.** $5x^2 - 3x + 8 = 0$ **3C.** $2x^2 + 11x + 15 = 0$

−151; no real roots 1; two real roots

★ indicates multi-step problem

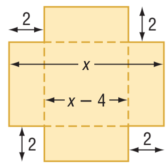
CHECK Your Understanding

Example 1 (pp. 493–494) **Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.**

1. $x^2 + 7x + 6 = 0$ **−6, −1** **2.** $t^2 + 11t = 12$ **−12, 1**

3. $r^2 + 10r + 12 = 0$ **−8.6, −1.4** **4.** $3v^2 + 5v + 11 = 0$ **∅**

Example 2 (p. 495) ★ **5. MANUFACTURING** A pan is to be formed by cutting 2-centimeter-by-2-centimeter squares from each corner of a square piece of sheet metal and then folding the sides. If the volume of the pan is to be 441 square centimeters, what should the dimensions of the original piece of sheet metal be?
about 18.8 cm by 18.8 cm



Example 3 (p. 496) **State the value of the discriminant for each equation. Then determine the number of real roots of the equation.**

6. $m^2 + 5m - 6 = 0$ **49; 2 real roots** **7.** $s^2 + 8s + 16 = 0$ **0; 1 real root** **8.** $2z^2 + z = -50$ **−399; no real roots**

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–20	1, 2
21, 22	3
23–28	4

Exercise Levels
A: 9–28
B: 29–39
C: 40–44

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

9. $v^2 + 12v + 20 = 0$ **10.** $3t^2 - 7t - 20 = 0$ **11.** $5y^2 - y - 4 = 0$

12. $x^2 - 25 = 0$ **−5, 5** **13.** $r^2 + 25 = 0$ **∅** **14.** $2x^2 + 98 = 28x$ **7**

15. $4s^2 + 100 = 40s$ **5** **16.** $2r^2 + r - 14 = 0$ **17.** $2n^2 - 7n - 3 = 0$

18. $5v^2 - 7v = 1$ **−0.1, 1.5** **19.** $11z^2 - z = 3$ **−0.5, 0.6** **20.** $2w^2 = -(7w + 3)$ **−3, −0.5**

21. GEOMETRY What are the dimensions of rectangle *ABCD*? **5 cm by 16 cm**

Rectangle *ABCD*	
perimeter	42 cm
area	80 cm^2

22. PHYSICAL SCIENCE A projectile is shot vertically up in the air from ground level. Its distance s, in feet, after t seconds is given by $s = 96t - 16t^2$. Find the values of t when s is 96 feet. **about 1.3 s and 4.7 s**

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

23. $x^2 + 3x - 4 = 0$ **24.** $y^2 + 3y + 1 = 0$ **25.** $4p^2 + 10p = -6.25$

26. $1.5m^2 + m = -3.5$ **−20; no real roots** **27.** $2r^2 = \frac{1}{2}r - \frac{2}{3}$ **$-\frac{61}{12}$, no real roots** **28.** $\frac{4}{3}n^2 + 4n = -3$ **0; 1 real root**

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

29. $1.34d^2 - 1.1d = -1.02$ **∅** **30.** $-2x^2 + 0.7x = -0.3$ **−0.3, 0.6**

31. $2y^2 - \frac{5}{4}y = \frac{1}{2}$ **−0.3, 0.9** **32.** $w^2 + \frac{2}{25} = \frac{3}{5}w$ **0.2, 0.4**

9. −10, −2
10. −1.7, 4
11. −0.8, 1
16. −2.9, 2.4
17. −0.4, 3.9
23. 25; 2 real roots
24. 5; 2 real roots
25. 0; 1 real root

3 Practice

Formative Assessment

Use Exercises 1–8 to check for understanding.

Use the chart at the bottom of this page to customize assignments for your students.

Odd/Even Assignments

Exercises 9–28 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Exercise Alert!

Use a Calculator Exercises 9–20, 29–32 and 39 ask students either to round their answers or to give approximations. You may wish to have students use a calculator for these exercises.

DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	9–28, 40–65	9–27 odd, 45, 46	10–28 even, 40–44, 47–65
OL Core	9–35 odd, 37–65	9–28, 45, 46	29–44, 47–65
AL Advanced /Pre-AP	29–62 (optional: 63–65)		

Study Guide and Intervention
CRM pp. 27–28 OL AL ELL

9-4 Study Guide and Intervention
Solving Quadratic Equations by Using the Quadratic Formula

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions of $ax^2 + bx + c = 0$, where $a \neq 0$

Example 1 **Solve $x^2 + 2x = 3$ by using the Quadratic Formula.**

Rewrite the equation in standard form.

$x^2 + 2x = 3$ Original equation
$x^2 + 2x - 3 = 3 - 3$ Subtract 3 from each side.
$x^2 + 2x - 3 = 0$ Simplify.

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)} = \frac{-2 \pm \sqrt{16}}{2}$$

$x = \frac{-2 + 4}{2}$ or $x = \frac{-2 - 4}{2}$
$= 1$ $= -3$

The solution set is $\{-3, 1\}$.

Example 2 **Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.**

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)} = \frac{6 \pm \sqrt{44}}{2}$$

$x = \frac{6 + \sqrt{44}}{2}$ or $x = \frac{6 - \sqrt{44}}{2}$
≈ 6.3 ≈ -0.3

The solution set is $\{-0.3, 6.3\}$.

Exercises

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 3x + 2 = 0$ **1, 2**
2. $m^2 - 8m = -16$ **4**
3. $16r^2 - 8r = -1$ $\frac{1}{4}$
4. $x^2 + 5x = 6$ **−6, 1**
5. $3x^2 + 2x = 8$ $-2, \frac{4}{3}$
6. $8x^2 - 8x - 5 = 0$ **−0.4, 1.4**
7. $-4c^2 + 19c = 21$ $\frac{7}{4}, 3$
8. $2p^2 + 6p = 5$ **−3.7, 0.7**
9. $48x^2 + 22x - 15 = 0$ $-\frac{5}{6}, \frac{3}{8}$
10. $8x^2 - 4x = 24$ $-\frac{3}{2}, 2$
11. $2p^2 + 5p = 8$ **−3.6, 1.1**
12. $8y^2 + 9y - 4 = 0$ **−1.5, 0.3**
13. $2x^2 + 9x + 4 = 0$ $-4, -\frac{1}{2}$
14. $8y^2 + 17y + 2 = 0$ $-2, -\frac{1}{8}$
15. $3z^2 + 5z - 2 = 0$ $-2, \frac{1}{3}$
16. $-2x^2 + 8x + 4 = 0$ **−0.4, 4.4**
17. $a^2 + 3a = 2$ **−3.6, 0.6**
18. $2y^2 - 6y + 4 = 0$ **1, 2**

Lesson 9-4

Practice
CRM p. 30 OL AL

9-4 Practice
Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $g^2 + 2g - 3 = 0$ **−3, 1**
2. $a^2 + 8a + 7 = 0$ **−7, −1**
3. $v^2 - 4v + 6 = 0$ $\varnothing$
4. $d^2 - 6d + 7 = 0$ **1.6, 4.4**
5. $2z^2 + 9z - 5 = 0$ $-5, \frac{1}{2}$
6. $2r^2 + 12r + 10 = 0$ **−5, −1**
7. $2b^2 - 9b = -12$ $\varnothing$
8. $2h^2 - 5h = 12$ $-1\frac{1}{2}, 4$
9. $3p^2 + p = 4$ $-1\frac{1}{3}, 1$
10. $3m^2 - 1 = -8m$ **−2.8, 0.1**
11. $4y^2 + 7y = 15$ $-3, 1\frac{1}{4}$
12. $1.6n^2 + 2n + 2.5 = 0$ $\varnothing$
13. $4.5k^2 + 4k - 1.5 = 0$ **−1.2, 0.3**
14. $\frac{1}{2}c^2 + 2c + \frac{3}{2} = 0$ **−3, −1**
15. $3w^2 - \frac{3}{4}w = \frac{1}{2}$ **−0.3, 0.6**

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

16. $a^2 + 8a + 16 = 0$ **0; 1 real root**
17. $c^2 + 3c + 12 = 0$ **−39; no real roots**
18. $2w^2 + 12w = -7$ **88; 2 real roots**
19. $2u^2 + 15u = -30$ **−15; no real roots**
20. $4n^2 + 9 = 12n$ **0; 1 real root**
21. $3g^2 - 2g = 3.5$ **46; 2 real roots**
22. $2.5k^2 + 3k - 0.5 = 0$ **14; 2 real roots**
23. $\frac{3}{4}d^2 - 3d = -4$ **−3; no real roots**
24. $\frac{1}{4}s^2 = -s - 1$ **0; 1 real root**

CONSTRUCTION **For Exercises 25 and 26, use the following information.**
A roofer tosses a piece of roofing tile from a roof onto the ground 30 feet below. He tosses the tile with an initial downward velocity of 10 feet per second.

25. Write an equation to find how long it takes the tile to hit the ground. Use the model for vertical motion, $H = -16t^2 + vt + h$, where H is the height of an object after t seconds, v is the initial velocity, and h is the initial height. (*Hint:* Since the object is thrown down, the initial velocity is negative.) $H = -16t^2 - 10t + 30$

26. How long does it take the tile to hit the ground? **about 1.1 s**

27. **PHYSICS** Lupe tosses a ball up to Quyen, waiting at a third-story window, with an initial velocity of 30 feet per second. She releases the ball from a height of 6 feet. The equation $h = -16t^2 + 30t + 6$ represents the height h of the ball after t seconds. If the ball must reach a height of 25 feet for Quyen to catch it, does the ball reach Quyen? Explain. (*Hint:* Substitute 25 for h and use the discriminant.) **No; the discriminant, −316, is negative, so there is no solution.**

Word Problem Practice
CRM p. 31 OL AL

9-4 Word Problem Practice
Solving Quadratic Equations by Using the Quadratic Formula

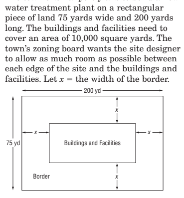
1. **BUSINESS** Tanya runs a catering business. Based on her records, her weekly profit can be approximated by the function $f(x) = x^2 + 2x - 37$, where x is the number of meals she caters. If $f(x)$ is negative, it means that the business has lost money. What is the least number of meals that Tanya needs to cater in order to have a profit? **6 meals**

2. **AERONAUTICS** At liftoff, the space shuttle *Discovery* has a constant acceleration of 16.4 feet per second squared and an initial velocity of 1341 feet per second due to the rotation of Earth. The distance *Discovery* has traveled t seconds after liftoff is given by the equation $d(t) = 1341t + 8.2t^2$. How long after liftoff has *Discovery* traveled 40,000 feet? Round your answer to the nearest tenth. **25.8 seconds**

3. **ARCHITECTURE** The Golden Ratio appears in the design of the Greek Parthenon because the width and height of the façade are related by the equation $\frac{W + H}{W} = \frac{W}{H}$. If the height of a model of the Parthenon is 16 inches, what is its width? Round your answer to the nearest tenth. **25.9 in.**

4. **CRAFTS** Madelyn cut a 60-inch pipe cleaner into two unequal pieces, and then she used each piece to make a square. The sum of the areas of the squares was 117 square inches. Let x = the length of one piece. Write and solve an equation to represent the situation and find the lengths of the two original pieces. $\left(\frac{60 - x}{4}\right)^2 + \left(\frac{x}{4}\right)^2 = 117$ **24 in. and 36 in.**

SITE DESIGN **For Exercises 5–7, refer to the following information. Round your answers to the nearest tenth.**
The town of Smallport plans to build a new water treatment plant on a rectangular piece of land 75 yards wide and 200 yards long. The buildings and facilities need to cover an area of 10,000 square yards. The town's zoning board wants the site designer to allow as much room as possible between each edge of the site and the buildings and facilities. Let x = the width of the border.



5. Use an equation similar to area = length x width to represent the situation. $10{,}000 = (200 - 2x)(75 - 2x)$

6. Write the equation in standard quadratic form and solve for x. $4x^2 - 550x + 5000 = 0$; **9.8 and 127.7**

7. What should be the width of the border? **9.79 yd**

Lesson 9-4

Real-World Link

Downhill skiing is the most popular type of snow skiing. Skilled skiers can obtain speeds of about 60 miles per hour as they race down mountain slopes.

Source: *World Book Encyclopedia*

Without graphing, determine the x-intercepts of the graph of each function.

33. $f(x) = 4x^2 - 9x + 4$ **about 0.6 and 1.6**

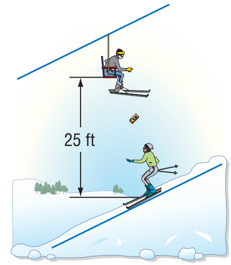
34. $f(x) = 13x^2 - 16x - 4$ **about −0.2 and 1.4**

Without graphing, determine the number of x-intercepts of the graph of each function.

35. $f(x) = 7x^2 - 3x - 1$ **2**

36. $f(x) = x^2 + 4x + 7$ **0**

RECREATION **For Exercises 37 and 38, use the following information.**



As Darius is skiing down a ski slope, Jorge is on the chairlift on the same slope. The chair lift has stopped. Darius stops directly below Jorge and attempts to toss a disposable camera up to him. If the camera is thrown with an initial velocity of 35 feet per second, the equation for the height of the camera is $h = -16t^2 + 35t + 5$, where h represents the height in feet and t represents the time in seconds.

★ 37. If the chairlift is 25 feet above the ground, will Jorge have 0, 1, or 2 chances to catch the camera? **0**

38. If Jorge is unable to catch the camera, when will it hit the ground? **about 2.3 s**

EXTRA PRACTICE
See pages 736, 752.
Math Online
Self-Check Quiz at ca.algebra1.com

39. **AMUSEMENT PARKS** The Demon Drop ride at Cedar Point takes riders to the top of a tower and drops them 60 feet at speeds reaching 80 feet per second. A function that models this ride is $h = -16t^2 + 64t - 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet? **about 2.5 s**

H.O.T. Problems

40. Let $a = 3$, $b = -2$, and $c = -4$. Substituting into the Quadratic Formula yields $\frac{2 \pm \sqrt{52}}{6}$. The roots of the function are $\frac{2 + \sqrt{52}}{6}$ and $\frac{2 - \sqrt{52}}{6}$.

41. Juanita; you must first write the equation in the form $ax^2 + bx + c = 0$ to determine the values of a, b, and c. Therefore, the value of c is −2, not 2.

40. **REASONING** Use the Quadratic Formula to show that $f(x) = 3x^2 - 2x - 4$ has two real roots.

41. **FIND THE ERROR** Lakeisha and Juanita are determining the number of solutions of $5y^2 - 3y = 2$. Who is correct? Explain your reasoning.

Lakeisha
$5y^2 - 3y = 2$
$b^2 - 4ac = (-3)^2 - 4(5)(2)$
$= -31$
Since the discriminant is negative, there are no real solutions.

Juanita
$5y^2 - 3y = 2$
$5y^2 - 3y - 2 = 0$
$b^2 - 4ac = (-3)^2 - 4(5)(-2)$
$= 49$
Since the discriminant is positive, there are two real roots.

42. **OPEN ENDED** Write a quadratic equation with no real solutions. Explain how you know there are no solutions. **Sample answer: $x^2 - x + 5 = 0$; the discriminant is −19, so the equation has no real solutions.**

43. **REASONING** Use factoring techniques to determine the number of real roots of the function $f(x) = x^2 - 8x + 16$. Compare this method to using the discriminant. **See Ch. 9 Answer Appendix.**

44. ***Writing in Math*** Describe three different ways to solve $x^2 - 2x - 15 = 0$. Which method do you prefer and why? **See Ch. 9 Answer Appendix.**

Enrichment
CRM p. 32 OL AL

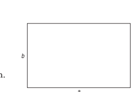
9-4 Enrichment

Golden Rectangles

A **golden rectangle** has the property that its sides satisfy the following proportion.

$$\frac{a + b}{a} = \frac{a}{b}$$

Two quadratic equations can be written from the proportion. These are sometimes called **golden quadratic** equations.



1. In the proportion, let $a = 1$. Use cross-multiplication to write a quadratic equation. $b^2 + b - 1 = 0$

2. Solve the equation in Exercise 1 for b. $b = \frac{-1 + \sqrt{5}}{2}$

BL = Below Grade Level
OL = On Grade Level
AL = Above Grade Level
ELL = English Language Learner

Additional pages not shown:

CRM *Lesson Reading Guide,* p. 26 BL OL ELL
CRM *Skills Practice,* p. 29 BL OL

STANDARDS PRACTICE 20.0, 7AF1.1

45. Which statement *best* describes why there is no real solution to the quadratic equation $y = x^2 - 6x + 13$? **C**

A The value of $(-6)^2 - 4 \cdot 1 \cdot 13$ is a perfect square.

B The value of $(-6)^2 - 4 \cdot 1 \cdot 13$ is equal to zero.

C The value of $(-6)^2 - 4 \cdot 1 \cdot 13$ is negative.

D The value of $(-6)^2 - 4 \cdot 1 \cdot 13$ is positive.

46. **REVIEW** In the system of equations $6x - 3y = 12$ and $2x + 5y = 9$, which expression can be correctly substituted for y in the equation $2x + 5y = 9$? **H**

F $12 + 2x$

G $12 - 2x$

H $-4 + 2x$

J $4 - 2x$

Spiral Review

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 9-3)

47. $x^2 - 8x = -7$ **1, 7**

48. $a^2 + 2a + 5 = 20$ **−5, 3**

49. $n^2 - 12n = 5$ **−0.4, 12.4**

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie. (Lesson 9-2)

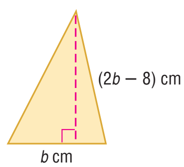
50. $x^2 - x = 6$

51. $2x^2 + x = 2$

52. $-x^2 + 3x + 6 = 0$

50–52. See margin.

53. **GEOMETRY** The triangle has an area of 96 square centimeters. Find the base b of the triangle. (Lesson 8-3) **12 cm**



Factor each polynomial. (Lesson 8-2)

54. $24r + 6s$ $\mathbf{6(4r + s)}$

55. $15xy^3 + y^4$ $\mathbf{y^3(15x + y)}$

56. $2ax + 6xc + ba + 3bc$ $\mathbf{(2x + b)(a + 3c)}$

Solve each inequality. Then check your solution. (Lesson 6-3)

57. $2m + 7 > 17$ $\{m \mid m > 5\}$

58. $-2 - 3x \geq 2$ $\left\{x \mid x \leq -1\frac{1}{3}\right\}$

59. $-20 \geq 8 + 7k$ $\{k \mid k \leq -4\}$

Write an equation of the line that passes through each point with the given slope. (Lesson 4-4)

60. $(2, 13)$, $m = 4$ $\mathbf{y = 4x + 5}$

61. $(-2, -7)$, $m = 0$ $\mathbf{y = -7}$

62. $(-4, 6)$, $m = \frac{3}{2}$ $\mathbf{y = \frac{3}{2}x + 12}$

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate $c(a^x)$ for each of the given values. (Lesson 1-1)

63. $a = 2, c = 1, x = 4$ **16**

64. $a = 7, c = 3, x = 2$ **147**

65. $a = 5, c = 2, x = 3$ **250**

Exercise Alert!

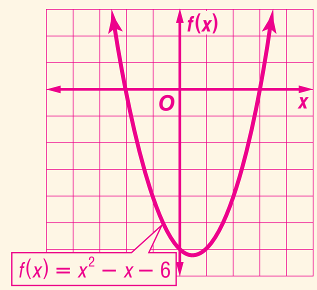
Find the Error Challenge students not only to find which student was incorrect, but also to explain what incorrect procedure led to the mistake.

4 Assess

Ticket Out the Door Make several copies each of five quadratic equations. Give one equation to each student. As students leave the room, ask them to tell you the discriminants of the equations and the number of real roots.

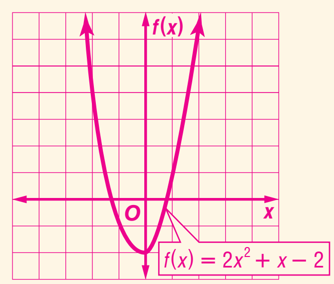
Additional Answers

50.



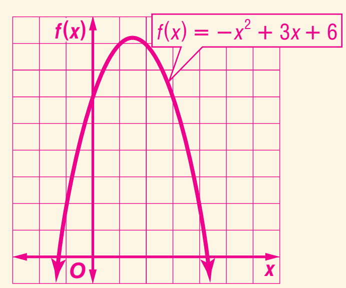
−2, 3

51.



$-2 < x < -1$, $0 < x < 1$

52.



$-2 < x < -1$, $4 < x < 5$

Pre-AP Activity Use after the Exercises

Since students are much more likely to retain a concept that they have researched and explained, ask them to research the derivation of the Quadratic Formula and to write short paragraphs showing and explaining the derivation.

Lesson 9-4 Solving Quadratic Equations by Using the Quadratic Formula 499

EXTEND 9-4

Lesson Notes

1 Focus

Materials for Each Group

- calculator

Teaching Tip

Remind students to look at the units in the question in order to determine whether to use 9.8 meters per second squared or 32 feet per second squared for the acceleration due to gravity.

2 Teach

Working in Cooperative Groups

Put students in pairs, mixing abilities. Have groups complete Exercises 1–5.

- Make sure students correctly use parentheses when typing the simplified quadratic formula solution into the calculator.

Ask:

- When does the velocity equal zero? When an object is dropped and not thrown.
- When is the velocity positive? When the object is projected upward.
- When is the velocity negative? When the object is projected downward.
- How does changing h affect the trajectory of an object? A change in h represents a change in height, a shift upward if h is positive and a shift downward if h is negative.

Practice Have students complete Exercises 1–4.

EXTEND 9-4

Algebra Lab
Applying Quadratic Equations

Standard 23.0 **Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.** (Key)

Many of the real-world problems you solved in Chapters 8 and 9 were physical problems involving the path of an object that is influenced by gravity. These paths, called **trajectories**, can be modeled by a quadratic function. The formula relating the height of the object $H(t)$ and time t is shown below.

$$H(t) = -\frac{1}{2}gt^2 + vt + h$$

Initial height of the object

Height of the object

Acceleration due to gravity

Initial velocity of the object

The acceleration due to gravity is 9.8 meters per second, per second; we express this by saying 9.8 meters per second squared. Similarly, it is 32 feet per second squared.

EXAMPLE 1

Juan kicks a football at a velocity of 25 meters per second. If the ball makes contact with his foot 0.5 meter off the ground, how long will the ball stay in the air?

We want to find the time t when $H(t)$ is 0. First substitute the known values into the motion formula. Since the known measures are written in terms of meters and meters per second, use 9.8 meters per second squared for the acceleration due to gravity.

$H(t) = -\frac{1}{2}gt^2 + vt + h$ — Motion Formula

$0 = -\frac{1}{2}(9.8)t^2 + 25t + 0.5$ — $H(t) = 0$, $g = 9.8$, $v = 25$, $h = 0.5$

$0 = -4.9t^2 + 25t + 0.5$ — Simplify.

Use the Quadratic Formula to solve for t.

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic Formula

$= \frac{-25 \pm \sqrt{25^2 - 4(-4.9)(0.5)}}{2(-4.9)}$ — $a = -4.9$, $b = 25$, $c = 0.5$

$= \frac{-25 \pm \sqrt{634.8}}{-9.8}$ — Simplify.

$t \approx -0.02$ or $t \approx 5.12$ — Use a calculator.

Since time cannot be a negative value, discard the negative solution. The football will be in the air about 5 seconds.

500 Chapter 9 Quadratic and Exponential Functions

If an object were projected downward, the initial velocity of the object is negative.

EXAMPLE 2

Katharine is on a bridge 12 feet above a pond. She throws a handful of fish food straight down with a velocity of 8 feet per second. In how many seconds will it reach the surface of the water?

Since the units given are in feet, use $g = 32$ ft/s^2. Katharine throws the food down, so the initial velocity is negative. When the food hits the water, $H(t)$ will be 0 feet.

$H(t) = -\frac{1}{2}gt^2 + vt + h$ Motion Formula

$0 = -\frac{1}{2}(32)t^2 - 8t + 12$ $H(t) = 0, g = 32, v = -8, h = 12$

$0 = -16t^2 - 8t + 12$ Simplify.

$0 = -4t^2 - 2t + 3$ Divide each side by 4.

Use the Quadratic Formula to solve for t.

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{2 \pm \sqrt{(-2)^2 - 4(-4)(3)}}{2(-4)}$ $a = -4, b = -2, c = 3$

$= \frac{2 \pm \sqrt{52}}{-8}$ Simplify.

$t \approx -1.15$ or $t \approx 0.65$ Use a calculator.

Discard the negative solution. The fish food will hit the water in 0.65 second.

Exercises

1. Darren swings at a golf ball on the ground with a velocity of 10 feet per second. How long was the ball in the air? **about 0.625 s**
2. Amalia hits a volleyball at a velocity of 15 meters per second. If the ball was hit from a height of 1.8 meters, determine the time it takes for the ball to land on the floor. Assume that the ball is not hit by another player. **about 3.2 s**
3. Michael is repairing the roof on a shed. He accidentally dropped a box of nails from a height of 14 feet. How long did it take for the box to land on the ground? Since the box was dropped and not thrown, $v = 0$. **about 0.94 s**
4. Carmen threw a penny into a fountain. She threw it from a height of 1.2 meters and at a velocity of 6 meters per second. How long did it take for the penny to hit the surface of the water? **about 0.17 s**

3 Assess

Formative Assessment

Use Exercise 4 to assess whether students can apply the quadratic equation to physical problems where the motion of an object is under the force of gravity.

From Concrete to Abstract What other real-world problems can be modeled by parabolas? Sample answer: designing arched doorways, landscaping patterns.

Extending the Concept

Ask: What factors other than gravity may affect the trajectory of an object? Sample answer: wind, pressure.

9-5 Lesson Notes

1 Focus

Standards Alignment

Before Lesson 9-5
Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems from Standard 7AF3.1

Lesson 9-5
Use the laws of fractional exponents in problems involving exponential growth and decay from Standard 2A12.0

After Lesson 9-5
Use the definition of logarithms to translate between logarithms in any base from Standard 2A13.0

2 Teach

Scaffolding Questions Have students read *Get Ready for the Lesson.*

Ask:

- What happens to the number of pliers in each level? It doubles.
- How many pliers would there be in a ninth level? In a tenth level? 512 in a ninth level; 1024 in a tenth
- Suppose Mr. Warther carved pliers for powers of 3. By how much would the number of pliers increase at each level? by three times
- How many pliers would be on the eighth level if each level were a power of three? 6561 pliers

9-5 Exponential Functions

Main Ideas

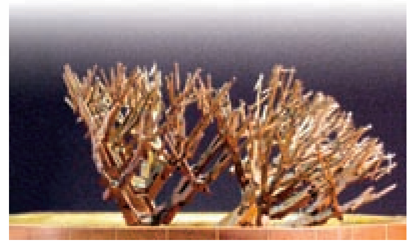
- Graph exponential functions.
- Identify data that displays exponential behavior.

Preparation for Algebra II Standard 12.0
Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay. (Key)

New Vocabulary

exponential function

GET READY for the Lesson



Earnest "Mooney" Warther was a whittler and a carver. For one of his most unusual carvings, Mooney carved a large pair of pliers in a tree.

From this original carving, he carved another pair of pliers in each handle of the original. Then he carved another pair of pliers in each of those handles. He continued this pattern to create the original pliers and 8 more layers of pliers. Even more amazing is the fact that all of the pliers work.

Graph Exponential Functions The number of pliers on each level is given in the table below.

Level	Number of Pliers	Power of 2
Original	1	2^0
First	1(2) = 2	2^1
Second	2(2) = 4	2^2
Third	2(2)(2) = 8	2^3
Fourth	2(2)(2)(2) = 16	2^4
Fifth	2(2)(2)(2)(2) = 32	2^5
Sixth	2(2)(2)(2)(2)(2) = 64	2^6
Seventh	2(2)(2)(2)(2)(2)(2) = 128	2^7
Eighth	2(2)(2)(2)(2)(2)(2)(2) = 256	2^8

Study the last column above. Notice that the exponent matches the level. So we can write an equation to describe y, the number of pliers for any given level x as $y = 2^x$. This function is neither linear nor quadratic. It is in the class of functions called **exponential functions** in which the variable is the exponent.

KEY CONCEPT *Exponential Function*

An exponential function is a function that can be described by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$.

As with other functions, you can use ordered pairs to graph an exponential function.

Lesson 9-5 Resources

Chapter 9 Resource Masters
Lesson Reading Guide, p. 33 BL OL ELL
Study Guide and Intervention, pp. 34–35 BL OL ELL
Skills Practice, p. 36 BL OL
Practice, p. 37 OL AL
Word Problem Practice, p. 38 OL AL
Enrichment, p. 39 OL AL
Spreadsheet, p. 40
Quiz 3, p. 52

Transparencies
5-Minute Check Transparency 9-5

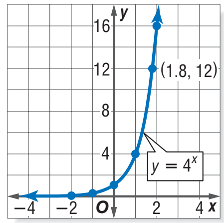
Additional Print Resources
Noteables™ Interactive Study Notebook with Foldables™
Teaching Algebra with Manipulatives

Technology
ca.algebra1.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

EXAMPLE 1 Graph an Exponential Function with $a > 1$

a. Graph $y = 4^x$. State the y-intercept.

x	4^x	y
-2	4^{-2}	$\frac{1}{16}$
-1	4^{-1}	$\frac{1}{4}$
0	4^0	1
1	4^1	4
2	4^2	16
3	4^3	64



Graph the ordered pairs and connect the points with a smooth curve. The y-intercept is 1.

b. Use the graph to determine the approximate value of $4^{1.8}$.

The graph represents all real values of x and their corresponding values of y for $y = 4^x$. So, the value of y is about 12 when $x = 1.8$. Use a calculator to confirm this value. $4^{1.8} \approx 12.12573253$

Study Tip

Exponential Graphs

Notice that the y-values change little for small values of x, but they increase quickly as the values of x become greater.

CHECK Your Progress

1A. Graph $y = 7^x$. State the y-intercept. **1; See margin for graph.**

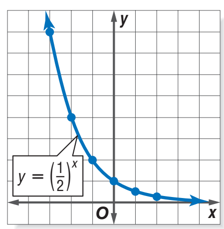
1B. Use the graph to determine the approximate value of $y = 7^{0.1}$ to the nearest tenth. Use a calculator to confirm the value. **1.2**

The graphs of functions of the form $y = a^x$, where $a > 1$, all have the same shape as the graph in Example 1, rising faster and faster as you move from left to right.

EXAMPLE 2 Graph Exponential Functions with $0 < a < 1$

a. Graph $y = \left(\frac{1}{2}\right)^x$. State the y-intercept.

x	$\left(\frac{1}{2}\right)^x$	y
-3	$\left(\frac{1}{2}\right)^{-3}$	8
-2	$\left(\frac{1}{2}\right)^{-2}$	4
-1	$\left(\frac{1}{2}\right)^{-1}$	2
0	$\left(\frac{1}{2}\right)^{0}$	1
1	$\left(\frac{1}{2}\right)^{1}$	$\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^{2}$	$\frac{1}{4}$



The y-intercept is 1. *Notice that the y-values decrease less rapidly as x increases.*

b. Use the graph to determine the approximate value of $\left(\frac{1}{2}\right)^{-2.5}$.

The value of y is about $5\frac{1}{2}$ when $x = -2.5$. Use a calculator to confirm this value. $\left(\frac{1}{2}\right)^{-2.5} \approx 5.656854249$

Math Online Extra Examples at ca.algebra1.com

Graph Exponential Functions

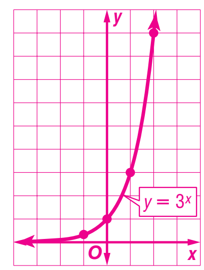
Example 1 shows how to graph an exponential function when a is greater than 1. **Example 2** shows how to graph an exponential function when a is greater than 0 and less than 1. **Example 3** shows how to use exponential functions to solve a real-world problem.

Formative Assessment

Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

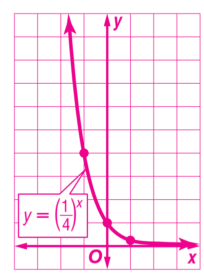
ADDITIONAL EXAMPLES

1 a. Graph $y = 3^x$. State the y-intercept. **1**



b. Use the graph to determine the approximate value of $3^{1.5}$. **about 5**

2 a. Graph $y = \left(\frac{1}{4}\right)^x$. State the y-intercept. **1**



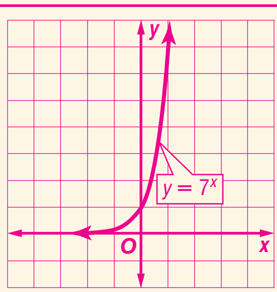
b. Use the graph to determine the approximate value of $\left(\frac{1}{4}\right)^{-1.5}$. **8**

Additional Examples are also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Additional Answer

1A.



Tips for New Teachers: Student Misconceptions

Make sure students understand that the graphs of exponential functions never actually touch the x-axis. It is acceptable for hand-drawn graphs to show the graph just above and about parallel to the x-axis as long as students understand that the graph gets infinitely closer to the axis without touching it.

Focus on Mathematical Content

Graphs of Exponential Functions An exponential function is a function that can be written in the form $y = a^x$, where a is greater than zero but not equal to one. When $a > 1$, $y = a^x$ increases as x increases. The greater the value of a, the more rapidly the function increases. When $0 < a < 1$, $y = a^x$ decreases as x increases. The smaller the value of a, the more rapidly the function increases.

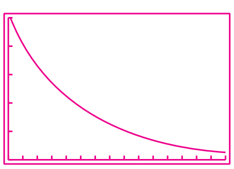
Algebra Lab

Discuss the similarities and differences among each set of graphs. Ask students to predict the effect of different changes to a function on the graph of the function.

ADDITIONAL EXAMPLE

3 **DEPRECIATION** Some people say that the value of a new car decreases as soon as it is driven off the lot. The function $V = 25{,}000 \cdot 0.82^t$ models the depreciation in the value of a new car that originally cost \$25,000. V represents the value of the car and t represents the time in years from the time of purchase.

a. Graph the function. What values of V and t are meaningful in the function?



Only values of $V \leq 25{,}000$ and $t \geq 0$ are meaningful.

b. What is the car's value after one year? about \$20,500

c. What is the car's value after five years? about \$9,270

CHECK Your Progress

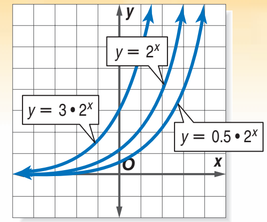
2A. Graph $y = \left(\frac{1}{3}\right)^x + 2$. State the y-intercept. **3; See margin for graph.**

2B. Use the graph to determine the approximate value of $y = \left(\frac{1}{3}\right)^{-1.5} + 2$ to the nearest tenth. Use a calculator to confirm the value. **7.2**

ALGEBRA LAB

Transformations of Exponential Functions

The graphs of $y = 2^x$, $y = 3 \cdot 2^x$, and $y = 0.5 \cdot 2^x$ are shown at the right. Notice that the y-intercept of $y = 2^x$ is 1, the y-intercept of $y = 3 \cdot 2^x$ is 3, and the y-intercept of $y = 0.5 \cdot 2^x$ is 0.5. The graph of $y = 3 \cdot 2^x$ is steeper than the graph of $y = 2^x$. The graph of $y = 0.5 \cdot 2^x$ is not as steep as the graph of $y = 2^x$.



THINK AND DISCUSS

Graph each set of equations on the same plane. Compare and contrast the graphs. **1–4. See Ch. 9 Answer Appendix.**

1. $y = 2^x$	**2.** $y = 2^x$	**3.** $y = 2^x$	**4.** $y = 3 \cdot 2^x$
$y = 2^x + 3$	$y = 2^{x+5}$	$y = 3^x$	$y = 3(2^x - 1)$
$y = 2^x - 4$	$y = 2^{x-4}$	$y = 5^x$	$y = 3(2^x + 1)$

Real-World Link

The first successful photographs of motion were made in 1877. Today, the motion picture industry is big business, with the highest-grossing movie making \$1,835,300,000.

Source: imdb.com

Real-World EXAMPLE — Use Exponential Functions to Solve Problems

3 **MOTION PICTURES** **Movie ticket sales decrease each weekend after an opening. The function $E = 49.9 \cdot 0.692^w$ models the earnings of a popular movie. In this equation, E represents earnings in millions of dollars and w represents the weekend number.**

a. Make a table. What values of E and w are meaningful in the context of the problem?

w	E
0	49.9
1	34.5308
2	23.8953136
3	16.535557
4	11.44260545
5	7.91828297

Only values where $E \leq 49.9$ and $w > 0$ are meaningful in the context of the problem.

b. How much did the movie make on the first weekend?

$E = 49.9 \cdot 0.692^w$ Original equation

$= 49.9 \cdot 0.692^1$ $w = 1$

$= 34.5308$ Use a calculator.

On the first weekend, the movie grossed about \$34.53 million.

504 Chapter 9 Quadratic and Exponential Functions

Intervention

Identifying Exponential Functions The LIST and STAT PLOT features of a graphing calculator can be used to graph data and determine whether functions are exponential. Enter the x-values into L1 and the corresponding y-values into L2. Turn on STAT PLOT and view the plotted points. After adjusting the window to fit the data, it should be possible to determine whether the data exhibits exponential behavior.

c. How much did it make on the fifth weekend?

$E = 49.9 \cdot 0.692^w$ Original equation

$= 49.9 \cdot 0.692^5$ $w = 5$

≈ 7.918282973 Use a calculator.

On the fifth weekend, the movie grossed about $7.92 million.

CHECK Your Progress

3. BIOLOGY A certain bacteria doubles every 20 minutes. How many will there be after 2 hours? **64**

Online **Personal Tutor at** ca.algebra1.com

Study Tip

Checking Answers

The graph of an exponential function may resemble part of the graph of a quadratic function. So be sure to check for a pattern as well as looking at a graph.

Identify Exponential Behavior How do you know if a set of data is exponential? One method is to observe the shape of the graph. Another way is to use the problem-solving strategy *look for a pattern*.

EXAMPLE Identify Exponential Behavior

4 **Determine whether the set of data at the right displays exponential behavior. Explain why or why not.**

x	0	10	20	30	40	50
y	80	40	20	10	5	2.5

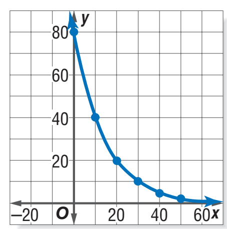
Method 1 Look for a Pattern

The domain values are at regular intervals of 10. Look for a common factor among the range values.

$$80 \underset{\times \frac{1}{2}}{\rightarrow} 40 \underset{\times \frac{1}{2}}{\rightarrow} 20 \underset{\times \frac{1}{2}}{\rightarrow} 10 \underset{\times \frac{1}{2}}{\rightarrow} 5 \underset{\times \frac{1}{2}}{\rightarrow} 2.5$$

Since the domain values are at regular intervals and the range values differ by a common factor, the data are probably exponential. Its equation may involve $\left(\frac{1}{2}\right)^x$.

Method 2 Graph the Data



The graph shows a rapidly decreasing value of y as x increases. This is a characteristic of exponential behavior.

CHECK Your Progress

4. Determine whether the set of data displays exponential behavior. Explain why or why not.

x	0	10	20	30	40	50
y	15	21	27	33	39	45

4. The domain values are at regular intervals of 10. The range values have a common difference 6. The data do not display exponential behavior, but rather linear behavior.

★ indicates multi-step problem

CHECK Your Understanding

Examples 1, 2 (pp. 503–504)

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value. **1–3. See margin.**

1. $y = 3^x; 3^{1.2}$ **2.** $y = \left(\frac{1}{4}\right)^x; \left(\frac{1}{4}\right)^{1.7}$ **3.** $y = 9^x; 9^{0.8}$

Graph each function. State the y-intercept. **4–5. See margin.**

4. $y = 2 \cdot 3^x$ **5.** $y = 4(5^x - 10)$

Identify Exponential Behavior

Example 4 shows how to determine whether a set of data displays exponential behavior.

ADDITIONAL EXAMPLE

4 Determine whether the set of data displays exponential behavior. Explain why or why not.

x	0	10	20	30
y	10	25	62.5	156.25

The domain values are at regular intervals, and the range values have a common factor of 2.5, so the set is probably exponential. Also, the graph shows rapidly increasing values of y as x increases.

3 Practice

Formative Assessment

Use Exercises 1–8 to check for understanding.

Use the chart at the bottom of the next page to customize assignments for your students.

Additional Answers

1–5. See Chapter 9 Answer Appendix for graphs.

1. 1; 3.7

2. 1; 0.1

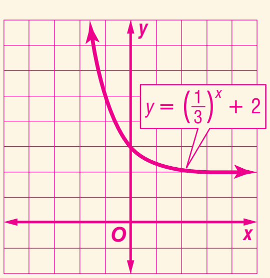
3. 1; 5.8

4. 2

5. –36

Additional Answer (Check Your Progress)

2A.



Odd/Even Assignments

Exercises 9–26 are structured so that students practice the same concepts whether they are assigned odd or even problems.

⚠ Exercise Alert!

Find the Error If students have trouble with Exercise 43, suggest that they multiply $\frac{1}{3} \times \frac{1}{3}$ and then $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$. Does the product increase or decrease as the exponent increases?

Additional Answers

6a. D: $\{d \mid d \geq 0\}$, the number of days is greater than or equal to 0; R: $\{y \mid y \geq 100\}$, the number of fruit flies is greater than or equal to 100.

7. Yes; the domain values are at regular intervals and the range values have a common factor 6.

8. No; the domain values are at regular intervals and the range values have a common difference 4.

9–18. See Chapter 9 Answer Appendix for graphs.

9. 1; 5.9

10. 1; 2.0

11. 1; 20

12. 1; 0.4

13. 1; 1.7

14. 1; 5.3

15. 5

16. 3

17. −6

18. 5

20. y-int is 300; There are 300 bacteria at 9:00 A.M. D: $\{t \mid t \geq 0\}$; R: $\{p \mid p \geq 300\}$.

22. See Chapter 9 Answer Appendix for graph; 12; $12 million in sales in 1995.

Example 3 (pp. 504–505)

6. BIOLOGY The function $f(t) = 100 \cdot 1.05^t$ models the growth of a fruit fly population, where $f(t)$ is the number of flies and t is time in days.

a. What values for the domain and range are reasonable in the context of this situation? Explain. **See margin.**

b. After two weeks, approximately how many flies are in this population? **about 198 fruit flies**

Example 4 (p. 505)

Determine whether the data in each table display exponential behavior. Explain why or why not. **7–8. See margin.**

7.

x	0	1	2	3	4	5
y	1	6	36	216	1296	7776

8.

x	4	6	8	10	12	14
y	5	9	13	17	21	25

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–18	1, 2
19–22	3
23–26	4

Exercise Levels
A: 9–26
B: 28–40
C: 41–47

23. No; the domain values are at regular intervals and the range values have a common difference 3.

24. Yes; the domain values are at regular intervals and the range values have a common factor 0.5.

25. Yes; the domain values are at regular intervals and the range values have a common factor 0.75.

26. No; the domain values are at regular intervals, but the range values do not have a positive common factor.

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value. **9–14. See margin.**

9. $y = 5^x$; $5^{1.1}$ **10.** $y = 10^x$; $10^{0.3}$ **11.** $y = \left(\frac{1}{10}\right)^x$; $\left(\frac{1}{10}\right)^{-1.3}$

12. $y = \left(\frac{1}{5}\right)^x$; $\left(\frac{1}{5}\right)^{0.5}$ **13.** $y = 6^x$; $6^{0.3}$ **14.** $y = 8^x$; $8^{0.8}$

Graph each function. State the y-intercept. **15–18. See margin.**

15. $y = 5(2^x)$ **16.** $y = 3(5^x)$ **17.** $y = 3^x - 7$ **18.** $y = 2^x + 4$

BIOLOGY For Exercises 19 and 20, use the following information.
A population of bacteria in a culture increases according to the model $p = 300 \cdot 2.7^{0.02t}$, where t is the number of hours and $t = 0$ corresponds to 9:00 A.M.

19. Use this model to estimate the number of bacteria at 11 A.M. **about 312**

★ **20.** Graph the function and name the y-intercept. Describe what the y-intercept represents and describe a reasonable domain and range for this situation. **See margin.**

BUSINESS For Exercises 21 and 22, use the following information.
The amount of money spent at West Outlet Mall in Midtown continues to increase. The total $T(x)$ in millions of dollars can be estimated by the function $T(x) = 12(1.12)^x$, where x is the number of years after it opened in 1995.

★ **21.** According to the function, find the amount of sales for the mall in the years 2005, 2006, and 2007.

★ **22.** Graph the function and name the y-intercept. What does the y-intercept represent in this problem? **See Ch. 9 Answer Appendix.**

21. about $37.27 million; about $41.74 million; about $46.75 million

Determine whether the data in each table display exponential behavior. Explain why or why not.

23.

x	−2	−1	0	1
y	−5	−2	1	4

24.

x	0	1	2	3
y	1	0.5	0.25	0.125

25.

x	10	20	30	40
y	16	12	9	6.75

26.

x	−1	0	1	2
y	−0.5	1.0	−2.0	4.0

27–29. See Ch. 9 Answer Appendix for graphs.

Graph each function. State the y-intercept.

27. $y = 2(3^x) - 1$ **1** **28.** $y = 2(3^x + 1)$ **4** **29.** $y = 3(2^x - 5)$ **−12**

DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	9–26, 41–43, 47–66	9–25 odd, 48, 49	10–26 even, 41–43, 47, 50–66
OL Core	9–35 odd, 36–43, 47–66	9–26, 48, 49	27–43, 47, 50–66
AL Advanced /Pre-AP	27–62 (optional: 63–66)		

506 Chapter 9 Quadratic and Exponential Functions

Real-World Link

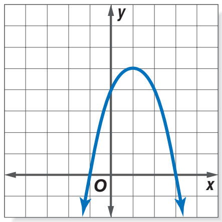
The first Boston Marathon was held in 1886. The distance of this race was based on the Greek legend that Pheidippides ran 24.8 miles from Marathon to Athens to bring the news of victory over the Persian army.

Source: www.bostonmarathon.org

Identify each function as *linear, quadratic,* or *exponential.*

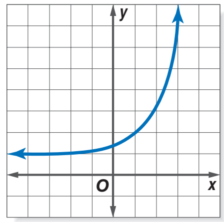
30. $y = 4^x + 3$ **exponential** **31.** $y = 2x(x - 1)$ **quadratic** **32.** $5x + y = 8$ **linear**

33.



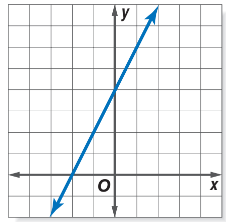
quadratic

34.



exponential

35.



linear

TOURNAMENTS For Exercises 36–38, use the following information.

In a quiz bowl competition, three schools compete, and the winner advances to the next round. Therefore, after each round, only $\frac{1}{3}$ of the schools remain in the competition for the next round. Suppose 729 schools start the competition.

36. Write an exponential function to describe the number of schools remaining after x rounds. **36.** $y = 729\left(\frac{1}{3}\right)^x$

37. How many schools are left after 3 rounds? **27 schools**

38. How many rounds will it take to declare a champion? **6 rounds**

ANALYZE TABLES For Exercises 39 and 40, use the following information.

A runner is training for a marathon, running a total of 20 miles per week on a regular basis. She plans to increase the distance $D(x)$ in miles according to the function $D(x) = 20(1.1)^x$, where x represents the number of weeks of training.

39. Copy and complete the table showing the number of miles she plans to run.

Week	Distance (miles)
1	22
2	24.2
3	26.62
4	29.282

40. The runner's goal is to work up to 50 miles per week. What is the first week that the total will be 50 miles or more? **10th week**

EXTRA PRACTICE
See pages 736, 752.

Math Online
Self-Check Quiz at ca.algebra1.com

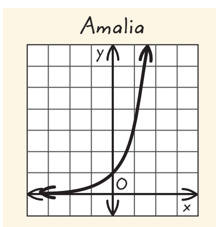
H.O.T. Problems

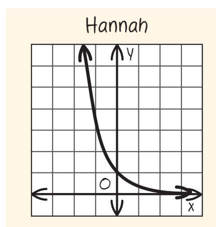
41. REASONING Determine whether the graph of $y = a^x$, where $a > 0$ and $a \neq 1$, *sometimes, always,* or *never* has an x-intercept. Explain your reasoning.
Never; the graph will never intersect the x-axis.

42. OPEN ENDED Choose an exponential function that represents a real-world situation and graph the function. Analyze the graph.
See Ch. 9 Answer Appendix.

43. FIND THE ERROR Amalia and Hannah are graphing $y = \left(\frac{1}{3}\right)^x$. Who is correct? Explain your reasoning.

Hannah; the graph of $y = \left(\frac{1}{3}\right)^x$ decreases as x increases.





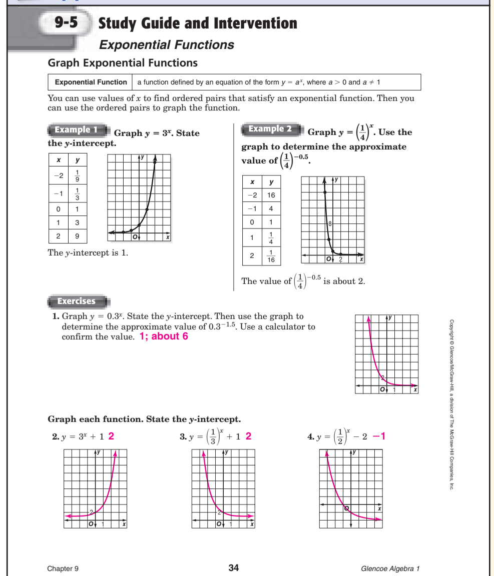
CHALLENGE Describe the graph of each equation as a transformation of the graph of $y = 5^x$.

44. $y = \left(\frac{1}{5}\right)^x$ **45.** $y = 5^x + 2$ **46.** $y = 5^x - 4$

44. a reflection over the y-axis
45. a translation 2 units up
46. a translation 4 units down

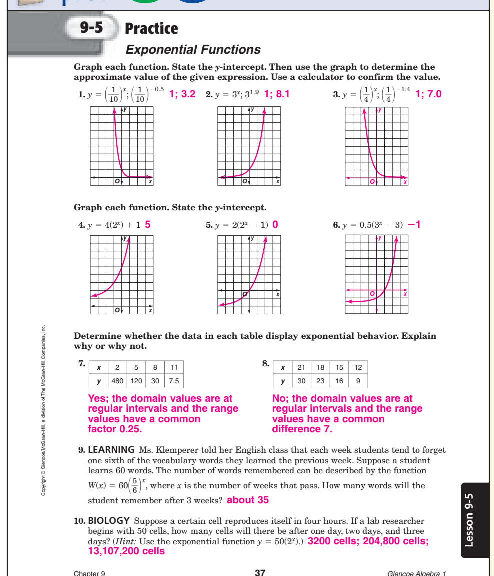
Study Guide and Intervention

CRM pp. 34–35 OL AL ELL


9-5 Study Guide and Intervention
Exponential Functions
Graph Exponential Functions

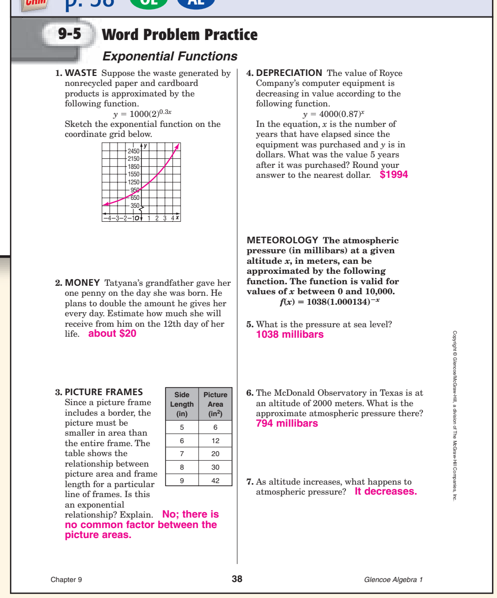
Practice

CRM p. 37 OL AL


9-5 Practice
Exponential Functions

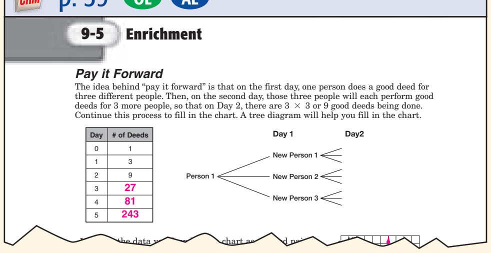
Word Problem Practice

CRM p. 38 OL AL


9-5 Word Problem Practice
Exponential Functions

Enrichment

CRM p. 39 OL AL


9-5 Enrichment
Pay it Forward

BL = *Below Grade Level*
OL = *On Grade Level*
AL = *Above Grade Level*
ELL = *English Language Learner*

Additional pages not shown:

CRM ***Lesson Reading Guide,*** p. 33 BL OL ELL
CRM ***Skills Practice,*** p. 36 BL OL

4 Assess

Crystal Ball Ask students to write how they think exponential functions will connect with the next lesson which involves growth and decay situations.

Formative Assessment

Check for student understanding of concepts in Lessons 9-4 and 9-5.

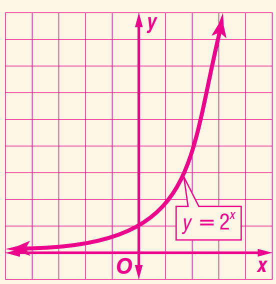
CRM Quiz 3, p. 52

FOLDABLES Study Organizer Foldables™ Follow-Up

At the end of this lesson, remind students to write notes and show examples on the page in their Foldables for this lesson.

Additional Answer

47. Sample answer: If the number of items on each level of a piece of art is a given number times the number of items on the previous level, an exponential function can be used to describe the situation. For the carving of the pliers, $y = 2^x$. For this situation, x is an integer between 0 and 8 inclusive. The values of y are 1, 2, 4, 8, 16, 32, 64, 128, and 256.



47. *Writing in Math* Use the information about the carving on page 502 to explain how exponential functions can be used in art. Include the exponential function representing the pliers, an explanation of which x and y values are meaningful, and the graph of this function. **See margin.**

STANDARDS PRACTICE 6NS1.3

48. Compare the graphs of $y = 2^x$ and $y = 6^x$. **A**

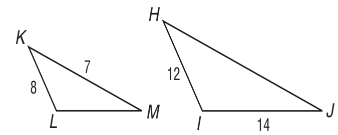
A The graph of $y = 6^x$ increases at a faster rate than the graph of $y = 2^x$.

B The graph of $y = 2^x$ increases at a faster rate than the graph of $y = 6^x$.

C The graph of $y = 6^x$ is the graph of $y = 2^x$ translated 4 units up.

D The graph of $y = 6^x$ is the graph of $y = 2^x$ translated 3 units up.

49. **REVIEW** $\triangle KLM$ is similar to $\triangle HIJ$. Which scale factor is used to transform $\triangle KLM$ to $\triangle HIJ$? **H**



F $\frac{1}{2}$

G 1

H $1\frac{1}{2}$

J 2

Spiral Review

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-4)

50. $x^2 - 9x - 36 = 0$ **−3, 12**

51. $2t^2 + 3t - 1 = 0$ **−1.8, 0.3**

52. $5y^2 + 3 = y$ **∅**

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 9-3)

53. $x^2 - 7x = -10$ **2, 5**

54. $a^2 - 12a = 3$ **−0.2, 12.2**

55. $t^2 + 6t + 3 = 0$ **−5.4, −0.6**

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 8-3)

56. $m^2 - 14m + 40$ **$(m - 4)(m - 10)$**

57. $t^2 - 2t + 35$ **prime**

58. $z^2 - 5z - 24$ **$(z - 8)(z + 3)$**

Solve each inequality. (Lesson 6-1)

59. $x + 7 > 2$ **$\{x \mid x > -5\}$**

60. $10 \geq x + 8$ **$\{x \mid x \leq 2\}$**

61. $y - 7 < -12$ **$\{y \mid y < -5\}$**

62. **NUMBER THEORY** Three times one number equals twice a second number. Twice the first number is 3 more than the second number. Find the numbers. (Lesson 5-4) **6, 9**

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate $p(1 + r)^t$ for each of the given values.** (Lesson 1-1)

63. $p = 5, r = \frac{1}{2}, t = 2$ **11.25**

64. $p = 300, r = \frac{1}{4}, t = 3$ **585.9375**

65. $p = 100, r = 0.2, t = 2$ **144**

66. $p = 6, r = 0.5, t = 3$ **20.25**

Pre-AP Activity Use as an Extension

Give students this scenario: a wise man asked his ruler to provide rice for his people. Rather than receiving a daily supply of rice, the wise man asked the ruler to give him 2 grains of rice for the first square on a chess board, 4 grains for the second, 8 for the third, 16 for the fourth, and so on, doubling the amount of rice with each square of the board.

Ask:

- "How many grains of rice will the wise man receive for the sixty-fourth square on the chessboard?" about 1.84×10^{19} grains
- "If one pound of rice has approximately 24,000 grains, how many tons of rice will the wise man receive on the last day?" (*Hint*: 1 ton = 2000 pounds) about 3.84×10^{11} tons

EXPLORE 9-6

Algebra Lab

Investigating Exponential Functions

Preparation for Algebra II Standard 12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay. (Key)

ACTIVITY

Step 1 Cut a sheet of notebook paper in half.

Step 2 Stack the two halves, one on top of the other.

Step 3 Make a table like the one at the right and record the number of sheets of paper you have in the stack after one cut.

Number of Cuts	Number of Sheets
0	1
1	2
2	4

Step 4 Cut the two stacked sheets in half, placing the resulting pieces in a single stack. Record the number of sheets of paper in the new stack after 2 cuts.

Step 5 Continue cutting the stack in half, each time putting the resulting piles in a single stack and recording the number of sheets in the stack. Stop when the resulting stack is too thick to cut.

Analyze the Results

1. Write a list of ordered pairs (x, y), where x is the number of cuts and y is the number of sheets in the stack. Notice that the list starts with the ordered pair $(0, 1)$, which represents the single sheet of paper before any cuts were made. **$(0, 1), (1, 2), (2, 4), (3, 8), (4, 16), \ldots$**
2. Continue the list beyond the point where you stopped cutting, until you reach the ordered pair for 7 cuts. Explain how you calculated the last y values for your list after you had stopped cutting.
3. Plot the ordered pairs in your list on a coordinate grid. Be sure to choose a scale for the y-axis so that you can plot all of the points.
4. Write a function that expresses y as a function of x. **$y = 2^x$**
5. Evaluate the function you wrote in Exercise 4 for $x = 8$ and $x = 9$. Does it give the correct number of sheets in the stack after 8 and 9 cuts?
6. Notebook paper usually stacks about 500 sheets to the inch. How thick would your stack of paper be if you had been able to make 9 cuts?
7. Suppose each cut takes about 5 seconds. If you had been able to keep cutting, you would have made 36 cuts in three minutes. At 500 sheets to the inch, make a conjecture as to how thick you think the stack would be after 36 cuts.
8. Calculate the thickness of your stack after 36 cuts. Write your answer in miles. **2169 mi**
9. Use the results of the Activity to complete a table like the one at the right for 0 to 7 cuts. Then write a function to describe the area y after x cuts.

Number of Cuts	Area of Sheet
0	1
1	0.5
2	
…	

2. $(5, 32), (6, 64), (7, 128)$; The y-value is found by raising 2 to the number of cuts.

3. See Ch. 9 Answer Appendix.

5. 256, 512; yes

6. about 1 in.

7. Sample answer: 1 million ft

9. $y = \left(\frac{1}{2}\right)^x$; See Ch. 9 Answer Appendix for table.

EXPLORE 9-6

Lesson Notes

1 Focus

Objective Use paper stacking to investigate an exponential function.

Materials

- scissors

Teaching Tip You may wish to do the Activity as a demonstration while students complete the table on the board.

2 Teach

Working in Cooperative Groups Place students in groups of 2 or 3, mixing abilities. Have groups complete the Activity and Exercises 1–4.

- Students may recognize that the y-value is doubled for each successive cut. Guide them to see that this can be written in the form 2^x.
- After students have plotted the ordered pairs in their lists, show them how to connect the points with a smooth curve rather than connecting each pair of points with a straight line.

Practice Have students complete Exercises 5–8.

3 Assess

Formative Assessment

Use Exercise 4 to assess whether students can write the function that models the activity.

From Concrete to Abstract

Ask: Which function increases more rapidly: $y = 2x$ or $y = 2^x$? Sample answer: $y = 2^x$ grows faster than $y = 2x$. If $x = 1000$, $y = 2^{1000}$ is clearly much larger than $y = 2 \cdot 1000 = 2000$.

9-6 Lesson Notes

1 Focus

Standards Alignment

Before Lesson 9-6
Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems from Standard 7AF3.1

Lesson 9-6
Use the laws of fractional exponents in problems involving exponential growth and decay from Standard 2A12.0

After Lesson 9-6
Use the definition of logarithms to translate between logarithms in any base from Standard 2A13.0

2 Teach

Scaffolding Questions Have students read *Get Ready for the Lesson*.
Ask:

- Looking at the graph, how do you know it is not linear? The y-value increases by the factor 1.137 each month. Since the values increase by a common factor, the function is exponential.
- Use the graph to predict the total number of blogs in millions in November 2005. About 54 million

9-6 Growth and Decay

Main Ideas

- Solve problems involving exponential growth.
- Solve problems involving exponential decay.

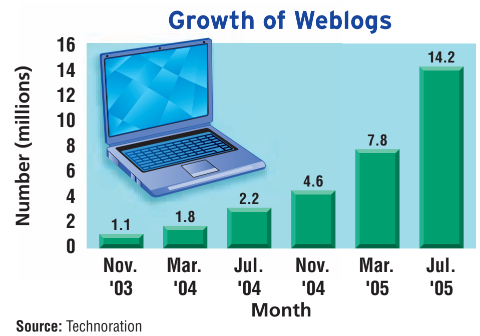
Preparation for Algebra II Standard 12.0
Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay. (Key)

New Vocabulary

exponential growth
compound interest
exponential decay

GET READY for the Lesson

The number of Weblogs or "blogs" increased at a monthly rate of about 13.7% between November 2003 and July 2005. Let y represent the total number of blogs in millions, and let t represent the number of months since November 2003. Then the average number per month can be modeled by $y = 1.1(1 + 0.137)^t$ or $y = 1.1(1.137)^t$.



Exponential Growth The equation for the number of blogs is in the form $y = C(1 + r)^t$. This is the general equation for **exponential growth** in which the initial amount C increases by the same percent over a given period of time.

KEY CONCEPT *General Equation for Exponential Growth*

The general equation for exponential growth is $y = C(1 + r)^t$ where y represents the final amount, C represents the initial amount, r represents the rate of change expressed as a decimal, and t represents time.

Real-World EXAMPLE Exponential Growth

1 **SPORTS** **In 1971, there were 294,105 females in high school sports. Since then, the number has increased an average of 8.5% per year.**

a. Write an equation to represent the number of females participating in high school sports since 1971.

$y = C(1 + r)^t$	General equation for exponential growth
$= 294{,}105(1 + 0.085)^t$	$C = 294{,}105$ and $r = 8.5\%$ or 0.085
$= 294{,}105(1.085)^t$	Simplify.

An equation to represent the number of females participating in high school sports is $y = 294{,}105(1.085)^t$, where y is the number of female athletes and t is the number of years since 1971.

Lesson 9-6 Resources

Chapter 9 Resource Masters
Lesson Reading Guide, p. 41 BL OL ELL
Study Guide and Intervention, pp. 42–43 BL OL ELL
Skills Practice, p. 44 BL OL
Practice, p. 45 OL AL
Word Problem Practice, p. 46 OL AL
Enrichment, p. 47 OL AL
Quiz 4, p. 52

Transparencies
5-Minute Check Transparency 9-6

Additional Print Resources
Noteables™ Interactive Study Notebook with Foldables™
Teaching Algebra with Manipulatives

Technology
ca.algebra1.com
Interactive Classroom CD-ROM
AssignmentWorks CD-ROM
Graphing Calculator Easy Files

b. **According to the equation, how many females participated in high school sports in 2005?**

$y = 294{,}105(1.085)^t$ — Equation for females participating in sports

$= 294{,}105(1.085)^{34}$ — $t = 2005 - 1971$ or 34

$\approx 4{,}711{,}004$ — Use a calculator.

In 2005, about 4,711,004 females participated.

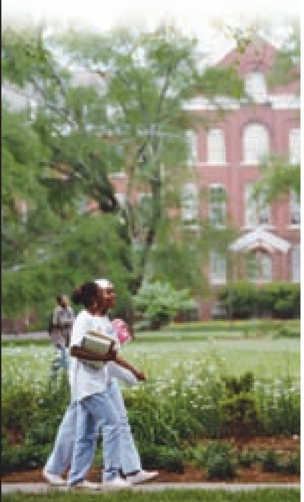
CHECK Your Progress 1A. $C = 18.9(1.19)^t$

TECHNOLOGY **Computer use has risen 19% annually since 1980.**

1A. If 18.9 million computers were in use in 1980, write an equation for the number of computers in use for t years after 1980.

1B. Predict the number of computers in 2015. about 8329.24 million computers

Online Personal Tutor at ca.algebra1.com



One special application of exponential growth is **compound interest.** The equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the current amount of the investment, P is the principal (initial amount of the investment), r represents the annual rate of interest expressed as a decimal, n represents the number of times the interest is compounded each year, and t represents the number of years that the money is invested.

Real-World EXAMPLE — Compound Interest

2 **COLLEGE** **Maria's parents invested \$14,000 at 6% per year compounded monthly. How much money will there be in 10 years?**

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ — Compound interest equation

$= 14{,}000\left(1 + \frac{0.06}{12}\right)^{12(10)}$ — $P = 14{,}000$, $r = 6\%$ or 0.06, $n = 12$, and $t = 10$

$= 14{,}000(1.005)^{120}$ — Simplify.

$\approx 25{,}471.55$ — Use a calculator.

There will be about \$25,471.55.

Real-World Link

According to the College Board, the 2004–2005 average costs for college were \$14,640 for students attending 4-year public colleges and \$30,295 for students at 4-year private colleges.

Source: *World Book Encyclopedia*

CHECK Your Progress

2. **MONEY** Determine the amount of an investment if \$300 is invested at an interest rate of 3.5% compounded monthly for 22 years.

about \$647.20

Exponential Decay A variation of the growth equation can be used as the general equation for exponential decay. In **exponential decay,** the original amount decreases by the same percent over a period of time.

KEY CONCEPT — *General Equation for Exponential Decay*

The general equation for exponential decay is $y = C(1 - r)^t$, where y represents the final amount, C represents the initial amount, r represents the rate of decay expressed as a decimal, and t represents time.

Math Online Extra Examples at ca.algebra1.com

Focus on Mathematical Content

Compound Interest In contrast to simple interest, compound interest is applied to the original principal and any previously earned interest. There are four ways to increase the amount in a compound-interest account: the investor can increase the initial principal, increase the annual interest rate, increase the number of compoundings per year, or increase the time that the money is in the account.

Exponential Growth

Example 1 shows how to solve a real-world problem involving exponential growth. **Example 2** shows how to solve a real-world problem involving compound interest.

Formative Assessment

Use the Check Your Progress exercises after each example to determine students' understanding of concepts.

ADDITIONAL EXAMPLES

1 **POPULATION** In 2005 the town of Flat Creek had a population of about 280,000 and a growth rate of 0.85% per year.

a. Write an equation to represent the population of Flat Creek since 2005. $y = 280{,}000\ (1.0085)^t$

b. According to the equation, what will be the population of Flat Creek in the year 2015? about 304,731

2 **COLLEGE** When Jing May was born, her grandparents invested \$1000 in a fixed rate savings account at a rate of 7% compounded annually. The money will go to Jing May when she turns 18 to help with her college expenses. What amount of money will Jing May receive from the investment? She will receive about \$3380.

Additional Examples are also in:

- Noteables™ Interactive Study Notebook with Foldables™
- Interactive Classroom PowerPoint® Presentations

Exponential Decay

Example 3 shows how to solve a real-world problem involving exponential decay.

ADDITIONAL EXAMPLE

3 **CHARITY** During an economic recession, a charitable organization found that its donations dropped by 1.1% per year. Before the recession, its donations were $390,000.

a. Write an equation to represent the charity's donations since the beginning of the recession. $A = 390{,}000(0.989)^t$

b. Estimate the amount of the donations 5 years after the start of the recession. about $369,017

Student Misconceptions

Tips for New Teachers

Remind students that in growth and decay equations, the amount inside the parentheses will be greater than for growth and less than for decay.

Real-World EXAMPLE — Exponential Decay

3 **SWIMMING** **A fully inflated raft loses 6.6% of its air every day. The raft originally contains 4500 cubic inches of air.**

a. Write an equation to represent the loss of air.

$y = C(1 - r)^t$ — General equation for exponential decay

$= 4500(1 - 0.066)^t$ — $C = 4500$ and $r = 6.6\%$ or 0.066

$= 4500(0.934)^t$ — Simplify.

An equation to represent the loss of air is $y = 4500(0.934)^t$, where y represents the amount of air in the raft in cubic inches and t represents the number of days.

b. Estimate the amount of air that will be lost after 7 days.

$y = 4500(0.934)^t$ — Equation for air loss

$= 4500(0.934)^7$ — $t = 7$

≈ 2790 — Use a calculator.

The amount of air lost after 7 days will be about 2790 cubic inches.

CHECK Your Progress 3A. $y = 776{,}677(1 - 0.01)^t$

POPULATION **During the past several years, the population of San Francisco County, California, has been decreasing at an average rate of about 1% per year. In 2000, its population was 776,677.**

3A. Write an equation to represent the population since 2000.

3B. If the trend continues, predict the population in 2010. about 702,413

★ indicates multi-step problem

CHECK Your Understanding

Example 1 (pp. 510–511)

ANALYZE GRAPHS **For Exercises 1 and 2, use the graph at the right and the following information.**
The median house price in the United States increased an average of 8.9% each year between 2002 and 2004. Assume this pattern continues.

Median House Price	
2002	$187,600
2003	$195,000
2004	$221,000

Source: RealEstateJournal.com

1. Write an equation for the median house price for t years after 2004.

2. Predict the median house price in 2009. about $338,479

1. $I = 221{,}000(1.089)^t$

Example 2 (p. 511)

3. **INVESTMENTS** Determine the amount of an investment if $400 is invested at an interest rate of 7.25% compounded quarterly for 7 years. about $661.44

Example 3 (p. 512)

★ **4.** **POPULATION** In 1995, the population of West Virginia reached 1,821,000, its highest in the 20th century. During the rest of the 20th century, its population decreased 0.2% each year. If this trend continues, predict the population of West Virginia in 2010. about 1,767,128 people

3 Practice

Formative Assessment

Use Exercises 1–4 to check for understanding.

Use the chart at the bottom of this page to customize assignments for your students.

Odd/Even Assignments

Exercises 6–13 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Real-World Connections

For Exercise 12, tell students that although industry experts believe music cassettes will not be around much longer in the West, Turkey sells 88 million cassettes a year, and India sells 80 million. In fact, cassette sales account for 50% of music sales in these countries.

DIFFERENTIATED HOMEWORK OPTIONS

Level	Assignment	Two-Day Option	
BL Basic	6–12, 16–27	7–11 odd, 19, 20	6–12 even, 16–18, 21–27
OL Core	7–11 odd, 13–27	6–12, 19, 20	13–18, 21–27
AL Advanced /Pre-AP	13–27		

512 **Chapter 9** Quadratic and Exponential Functions

Exercises

HOMEWORK HELP	
For Exercises	See Examples
5–8	1
9, 10	2
11, 12	3

Exercise Levels
A: 5–12
B: 13–16
C: 17–18

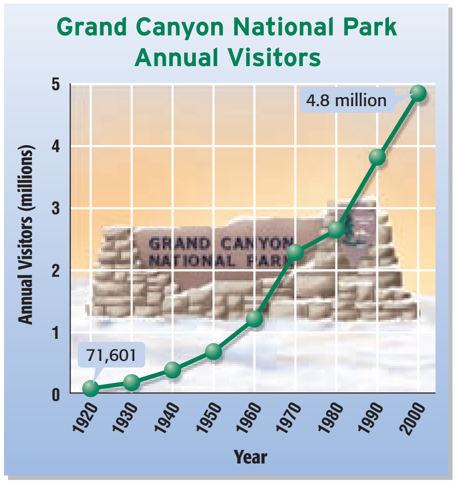
WEIGHT TRAINING **For Exercises 5 and 6, use the following information.**
In 1997, there were 43.2 million people who used free weights.

5. Assuming the use of free weights increases 6% annually, write an equation for the number of people using free weights t years from 1997. $W = 43.2(1.06)^t$

6. Predict the number of people using free weights in 2007. **about 77.36 million people**

★ **7.** **POPULATION** The population of Mexico has been increasing 1.7% annually. If the population was 100,350,000 in 2000, predict the population in 2012. **about 122,848,204 people**

★ **8.** **ANALYZE GRAPHS** The increase in the number of visitors to the Grand Canyon National Park is similar to an exponential function. If the average visitation has increased 5.63% annually since 1920, predict the number of visitors to the park in 2020. **about 17,125,650 visitors**



Source: Grand Canyon National Park

9. **INVESTMENTS** Determine the amount of an investment if \$500 is invested at an interest rate of 5.75% compounded monthly for 25 years. **about \$2097.86**

10. **INVESTMENTS** Determine the amount of an investment if \$250 is invested at an interest rate of 7.3% compounded quarterly for 40 years. **about \$4514.89**

★ **11.** **POPULATION** The country of Latvia has been experiencing a 1.1% annual decrease in population. In 2005, its population was 2,290,237. If the trend continues, predict Latvia's population in 2015. **about 2,050,422**

★ **12.** **MUSIC** In 1994, the sales of music cassettes reached its peak at \$2,976,400,000. Since then, cassette sales have been declining. If the annual percent of decrease in sales is 18.6%, predict the sales of cassettes in the year 2009. **about \$135,849,289**

13. about 128 g
14. about 76.36 g

ARCHAEOLOGY **For Exercises 13–15, use the following information.**
The *half-life* of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive Carbon-14 is found in all living organisms and has a half-life of 5730 years. Archaeologists use this information to estimate the age of fossils. Consider a living organism with an original Carbon-14 content of 256 grams. The number of grams remaining in the fossil of the organism after t years would be $256(0.5)^{\frac{t}{5730}}$.

13. If the organism died 5730 years ago, what is the amount of Carbon-14 today?

14. If an organism died 10,000 years ago, what is the amount of Carbon-14 today?

15. If the fossil has 32 grams of Carbon-14 remaining, how long ago did it live? (*Hint:* Make a table.) **about 17,190 years ago**

EXTRA PRACTICE
See pages 736, 752.
Math Online
Self-Check Quiz at
algebra1.com

Study Guide and Intervention
CRM pp. 42–43 OL AL ELL

9-6 Study Guide and Intervention
Growth and Decay

Exponential Growth Population increases and growth of monetary investments are examples of **exponential growth**. This means that an initial amount increases at a steady rate over time.

Exponential Growth	The general equation for exponential growth is $y = C(1 + r)^t$. • y represents the final amount. • C represents the initial amount. • r represents the rate of change expressed as a decimal. • t represents time.

Example 1 **POPULATION** **The population of Johnson City in 2000 was 25,000. Since then, the population has grown at an average rate of 3.2% each year.**

a. Write an equation to represent the population of Johnson City since 2000.
The rate 3.2% can be written as 0.032.
$y = C(1 + r)^t$
$y = 25{,}000(1 + 0.032)^t$
$y = 25{,}000(1.032)^t$

b. According to the equation, what will the population of Johnson City be in the year 2010?
In 2010, t will equal 2010 − 2000 or 10.
Substitute 10 for t in the equation from part a.
$y = 25{,}000(1.032)^{10}$ $t = 10$
$\approx 34{,}256$
In 2010, the population of Johnson City will be about 34,256.

Example 2 **INVESTMENT** **The Garcias have \$12,000 in a savings account. The bank pays 3.5% interest on savings accounts, compounded monthly. Find the balance in 3 years.**
The rate 3.5% can be written as 0.035.
The special equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the balance, P is the initial amount, r represents the annual rate expressed as a decimal, n represents the number of times the interest is compounded each year, and t represents the number of years the money is invested.
$A = P\left(1 + \frac{r}{n}\right)^{nt}$
$A = 12{,}000\left(1 + \frac{0.035}{12}\right)^{36}$
$A \approx 12{,}000(1.00292)^{36}$
$A \approx 13{,}328.09$
In three years, the balance of the account will be \$13,326.49.

Exercises

1. **POPULATION** The population of the United States has been increasing at an average annual rate of 0.91%. If the population of the United States was about 297,411,400 in the year 2005, predict the U. S. population in the year 2009. **Source:** U. S. Census Bureau **about 308,385,845**

2. **INVESTMENT** Determine the amount of an investment of \$2500 if it is invested at in interest rate of 5.25% compounded monthly for 4 years. **\$3082.78**

3. **POPULATION** It is estimated that the population of the world is increasing at an average annual rate of 1.3%. If the population of the world was about 6,472,416,997 in the year 2005, predict the world population in the year 2012. **Source:** U. S. Census Bureau **about 7,084,881,769**

4. **INVESTMENT** Determine the amount of an investment of \$100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years. **\$185,888.87**

Practice
CRM p. 45 OL AL

9-6 Practice
Growth and Decay

Lesson 9-6

COMMUNICATIONS **For Exercises 1 and 2, use the following information.**
Sports radio stations numbered 220 in 1996.

1. Write an equation for the number of sports radio stations for t years after 1996. $R = 220(1.143)^t$
2. If the trend continues, predict the number of sports radio stations in this format for the year 2010. **about 1429 stations**
3. **INVESTMENTS** Determine the amount of an investment if \$500 is invested at an interest rate of 4.25% compounded quarterly for 12 years. **\$830.41**
4. **INVESTMENTS** Determine the amount of an investment if \$300 is invested at an interest rate of 6.75% compounded semiannually for 20 years. **\$1131.73**
5. **HOUSING** The Greens bought a condominium for \$110,000 in 2005. If its value appreciates at an average rate of 6% per year, what will the value be in 2010? **about \$147,205**

DEFORESTATION **For Exercises 6 and 7, use the following information.**
During the 1990s, the forested area of Guatemala decreased at an average rate of 1.7%.

6. If the forested area in Guatemala in 1990 was about 34,400 square kilometers, write an equation for the forested area for t years after 1990. $C = 34{,}400(0.983)^t$
7. If this trend continues, predict the forested area in 2015. **about 22,407.65 km^2**
8. **BUSINESS** A piece of machinery valued at \$25,000 depreciates at a steady rate of 10% yearly. What will the value of the piece of machinery be after 7 years? **about \$11,957**
9. **TRANSPORTATION** A new car costs \$18,000. It is expected to depreciate at an average rate of 12% per year. Find the value of the car in 8 years. **about \$6473**
10. **POPULATION** The population of Osaka, Japan declined at an average annual rate of 0.05% for the five years between 1995 and 2000. If the population of Osaka was 11,013,000 in 2000 and it continues to decline at the same rate, predict the population in 2050. **about 10,741,000**

Word Problem Practice
CRM p. 46 OL AL

9-6 Word Problem Practice
Growth and Decay

1. **DEPRECIATION** The value of a new plasma television depreciates by about 7% each year. Aeryn purchases a 50-inch plasma television for \$5000. What is its value after 4 years? Round your answer to the nearest hundred. **about \$3700**
2. **MONEY** Hans opens a savings account by depositing \$1200 in an account that earns 3 percent interest compounded weekly. How much will his investment be worth in 10 years? Assume that there are exactly 52 weeks in a year and round your answer to the nearest cent. **\$1619.69**
3. **HIGHER EDUCATION** The table lists the average costs of attending a four-year college in the United States during the 2005–2006 school years.

College Sector	Tuition and Fees	Room and Board
Four-year Public	\$5,491	\$6,636
Four-year Private	\$21,235	\$7,791

Source: www.collegeboard.com

Russ's parents invested money in a savings account earning an average of 4.5 percent interest, compounded monthly. After 15 years, they have exactly the right amount to cover the tuition, fees, room and board for Russ's first year at a public college. What was their initial investment? Round your answer to the nearest dollar. **\$6182**

4. **POPULATION** In the 2000 U.S. Census, the population of the United States was estimated at 281 million. The annual rate of growth is about 1.1%. At this rate, what is the expected population at the time of the 2020 census? Round your answer to the nearest ten million. **350 million**

MEDICINE **For Exercises 5–7, use the following information. If necessary, round your answers to the nearest tenth.**
When doctors prescribe medication, they have to consider the rate at which the body filters a drug from the bloodstream. Suppose it takes the human body 6 days to filter out half of the Flu-B-Gone vaccine. The amount of Flu-B-Gone vaccine remaining in the bloodstream x hours after an injection is given by the equation $y = y_0(0.5)^t$, where y_0 is the initial amount. Suppose a doctor injects a patient with 20 μg (micrograms) of Flu-B-Gone.

5. How much of the vaccine will remain after 1 day? **17.8 μg**
6. How much of the vaccine will remain after 12 days? **5 μg**
7. After how many days will the amount of vaccine be less than 1 μg? **after 26 days**

Enrichment
CRM p. 47 OL AL

9-6 Enrichment

Lesson 9-6

Growth and Decay
Sierpinski Triangle is an example of a fractal that changes exponentially. Start with an equilateral triangle and find the midpoints of each side. Then connect the midpoints to form a smaller triangle. Remove this smaller triangle from the larger one.

Repeat the process to create the next triangle in the sequence. Find the midpoints of the sides of the three remaining triangles and connect them to form smaller triangles to be removed.

BL = *Below Grade Level*
OL = *On Grade Level*
AL = *Above Grade Level*
ELL = *English Language Learner*

Additional pages not shown:

CRM ***Lesson Reading Guide,*** p. 41 BL OL ELL
CRM ***Skills Practice,*** p. 44 BL OL

Lesson 9-6 Growth and Decay **513**

⚠ Exercise Alert!

Exercises 16 requires students to use the Internet or other resources.

4 Assess

Ticket Out the Door Make several copies each of five equations for exponential growth or decay. Give one equation to each student. As students leave the room, ask them to tell you whether their equations are for growth or decay.

Formative Assessment

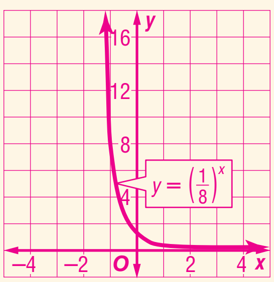
Check for student understanding of concepts in Lesson 9-6.

CRM Quiz 4, p. 52

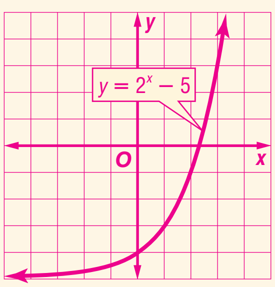
Additional Answers

17. Sample answer: Determine the amount of the investment if \$500 is invested at an interest rate of 7% compounded quarterly for 6 years.

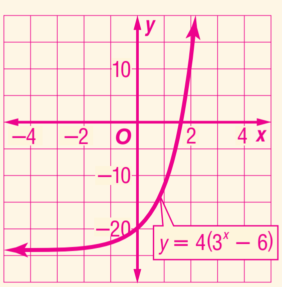
21.



22.



23.



16. RESEARCH Find the enrollment of your school district each year for the last decade. Find the rate of change from one year to the next. Then, determine the average annual rate of change for those years. Use this information to estimate the enrollment for your school district in ten years. **See students' work.**

H.O.T. Problems

17. OPEN ENDED Create a compound interest problem that could be solved by the equation $A = 500\left(1 + \frac{0.07}{4}\right)^{4(6)}$. **See margin.**

18. *Writing in Math* Use the information about Weblogs on page 510 to explain how exponential growth can be used to predict future blogs. Include an explanation of the equation $y = 1.1(1 + 0.137)^t$ and an estimate of the number of blogs in January 2010. **See Ch. 9 Answer Appendix.**

STANDARDS PRACTICE 7MG3.3

19. Lorena is investing a \$5000 inheritance from her aunt in a certificate of deposit that matures in 4 years. The interest rate is 6.25% compounded quarterly. What is the balance of the account after 4 years? **D**

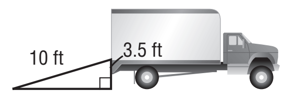
A \$5078.13

B \$5319.90

C \$5321.82

D \$6407.73

20. REVIEW Diego is building a 10-foot ramp for loading heavy equipment into the back of a semi-truck. If the floor of the truck is 3.5 feet off the ground, about how far from the truck should the ramp be? **F**



F 9 ft　　**H** 10.6 ft

G 10 ft　　**J** 11 ft

Spiral Review

Graph each function. State the *y*-intercept. (Lesson 9-5) **21–23. See margin for graphs.**

21. $y = \left(\frac{1}{8}\right)^x$ **1**　　**22.** $y = 2^x - 5$ **−4**　　**23.** $y = 4(3^x - 6)$ **−20**

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-4)

24. $m^2 - 9m - 10 = 0$ **−1, 10**　　**25.** $2t^2 - 4t = 3$ **−0.6, 2.6**　　**26.** $7x^2 + 3x + 1 = 0$ **∅**

27. SKIING A course for cross-country skiing is regulated so that the slope of any hill cannot be greater than 0.33. A hill rises 60 meters over a horizontal distance of 250 meters. Does the hill meet the requirements? Explain. (Lesson 4-1) **Yes; slope $= \frac{60}{250} = 0.24 < 0.33$.**

Cross-Curricular Project

Algebra and Science

Out of This World It is time to complete your project. Use the infomation and data you have gathered about the solar system to prepare a brochure, poster, or Web page. Be sure to include the three graphs, tables, diagrams, or calculations in the presentation.

Math Online Cross-Curricular Project at ca.algebra1.com

Pre-AP Activity — Use after the Exercises

Ask students to write their own exponential growth or decay problems, using data from periodicals or the Internet. Have students share their problems with the class when they are complete.

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EXTEND 9-6

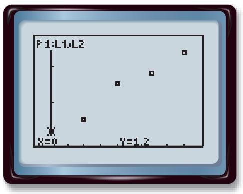
Graphing Calculator Lab
Curve Fitting

Concepts in Motion
Interactive Lab ca.algebra1.com

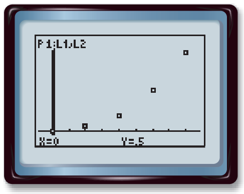
Preparation for Probability and Statistics Standard 4.0 Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families. (Key)

If there is a constant increase or decrease in data values, there is a linear trend. If the values are increasing or decreasing more and more rapidly, there may be a quadratic or exponential trend.

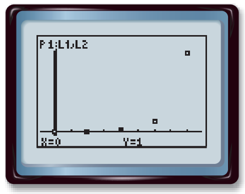
Linear Trend



Quadratic Trend



Exponential Trend



ACTIVITY 1

FARMING **A study is conducted in which groups of 25 corn plants are given a different amount of fertilizer and the gain in height after a certain time is recorded. The table below shows the results.**

Fertilizer (mg)	0	20	40	60	80
Gain in Height (in.)	6.48	7.35	8.73	9.00	8.13

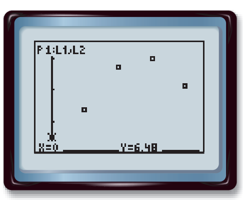
Step 1 Make a scatter plot.

- Enter the fertilizer in L1 and the height in L2.

KEYSTROKES: *Review entering a list on page 234.*

- Use STAT PLOT to graph the scatter plot.

KEYSTROKES: *Review statistical plots on page 234. Use* ZOOM *9 to graph.*



[−8, 88] scl: 5 by [6.0516, 9.4284] scl: 1

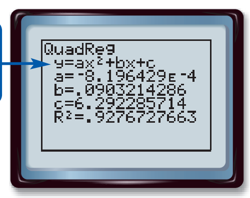
The graph appears to be a quadratic regression.

Step 2 Find the regression equation.

- Select DiagnosticOn from the CATALOG.
- Select QuadReg on the STAT CALC menu.

KEYSTROKES: STAT ▶ 5 ENTER

The equation is in the form $y = ax^2 + bx + c$.



The equation is about $y = -0.0008x^2 + 0.1x + 6.3$.

R^2 is the **coefficient of determination**. The closer R^2 is to 1, the better the model. To choose a quadratic or exponential model, fit both and use the one with the R^2 value closer to 1.

EXTEND 9-6

Lesson Notes

1 Focus

Objective Use a graphing calculator to find an appropriate regression equation for a set of data.

Materials

- graphing calculator

Teaching Tip Remind students that before entering data for a new problem into a list, they need to press STAT 4 ENTER to clear any previously entered data from particular lists. Clear previously entered equations from the Y=LIST by highlighting each equation and pressing the CLEAR key.

2 Teach

Working in Cooperative Groups
Put students in groups of 2 or 3, mixing abilities. Have groups complete the Activity and Exercises 1–4.

- In Step 1 of the Activity, make sure students clear previous lists before entering the data. Students should enter the amount of fertilizer in L1 and the height gain in L2.
- In Step 2 of the Activity, point out that the *a*-value, −8.196429E−4, is in scientific notation. This value corresponds to -8.196429×10^{-4}, or −0.0008196429.
- For Step 3 of the Activity, tell students that they must copy the quadratic regression exactly to the Y=LIST in order to get the proper graph.
- In Step 4, point out that to calculate the maximum for the function, press 2nd [CALC] 4. Remind students to set the left and right bounds far enough from the vertex to calculate the correct maximum.

Practice Have students complete Exercises 5–9.

3 Assess

Formative Assessment

In Step 4 of the Activity, the quadratic regression predicted that 55 milligrams produces the maximum height gain, while the data suggested that 60 milligrams would produce the greatest gain. Ask students to explain the difference. Sample answer: The quadratic regression equation is a best fit to data points that do not fall on an actual graphed function. There will be differences between actual data points and points that fall on the regression function.

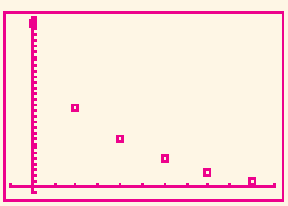
From Concrete to Abstract

Ask:

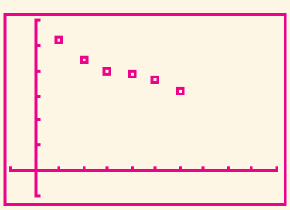
How do you determine whether to use a linear, quadratic, or exponential regression equation for your data? Sample answer: Make a scatter plot of your data points. If it looks close to a straight line, use a linear regression equation. If the data points follow a curve, fit a quadratic regression equation and an exponential regression equation to your points. The model with the coefficient of determination closest to 1 is the model to use.

Additional Answers

1. exponential; 0.9969724389



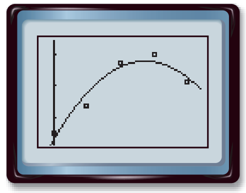
2. linear; 0.389164209



Step 3 Graph the regression equation.

- Copy the equation to the Y= list and graph.

KEYSTROKES: Y= VARS 5 ▶ ▶ 1 ZOOM 9



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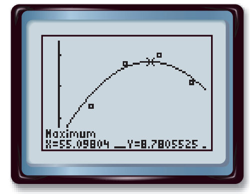
Step 4 Predict using the equation.

- Find the amount of fertilizer that produces the maximum gain in height.

KEYSTROKES: 2nd [CALC] 4



[−8, 88] scl: 5 by [6.0516, 9.4284] scl: 1

According to the graph, on average about 55 milligrams of the fertilizer produces the maximum gain.

Exercises

Plot each set of data points. Determine whether to use a *linear, quadratic,* or *exponential* regression equation. State the coefficient of determination.

1.

x	y
0.0	2.98
0.2	1.46
0.4	0.90
0.6	0.51
0.8	0.25
1.0	0.13

2.

x	y
1	25.9
2	22.2
3	20.0
4	19.3
5	18.2
6	15.9

3.

x	y
10	35
20	50
30	70
40	88
50	101
60	120

4.

x	y
1	3.67
3	5.33
5	6.33
7	5.67
9	4.33
11	2.67

1–4. See margin. **6.** $y = 0.326x^2 - 1299.082x + 1{,}295{,}315.229$; $r^2 = 0.995298$

TECHNOLOGY DVD players were introduced in 1997. For Exercises 5–8, use the table at the right.

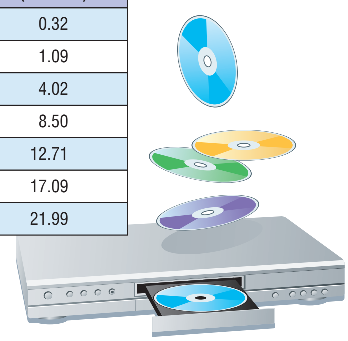
5. Make a scatter plot of the data. **See margin.**

6. Find an appropriate regression equation, and state the coefficient of determination.

7. Use the regression equation to predict the number of DVD players that will sell in 2008. **59.1 million**

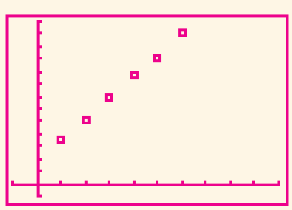
8. Do you believe your equation would be accurate for a year beyond the range of the data, such as 2020? Explain. **See margin.**

Year	DVD Players Sold (millions)
1997	0.32
1998	1.09
1999	4.02
2000	8.50
2001	12.71
2002	17.09
2003	21.99

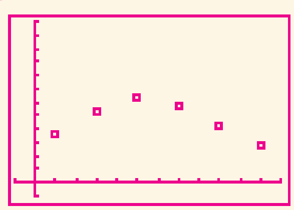


Source: Consumer Electronics Association

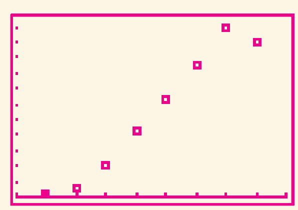
3. linear; 0.9974802029



4. quadratic; 0.97716799



5.



8. Sample answer: Yes; the price will start to increase using the quadratic model.

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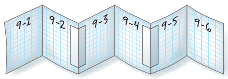
CHAPTER 9 Study Guide and Review

STUDY TO GO Download Vocabulary Review from ca.algebra1.com

FOLDABLES Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Graphing Quadratic Functions (Lesson 9-1)

- A quadratic function can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.
- The axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

Solving Quadratic Equations (Lessons 9-2, 9-3, and 9-4)

- The solutions of a quadratic equation are called the roots of the equation. They are the x-intercepts or zeros of the related quadratic function.
- Quadratic equations can be solved by completing the square. To complete the square for $x^2 + bx$, find $\frac{1}{2}$ of b, square this result, and then add the final result to $x^2 + bx$.
- The solutions of a quadratic equation can be found by using the Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Exponential Functions (Lessons 9-5 and 9-6)

- An exponential function can be described by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$.
- The general equation for exponential growth is $y = C(1 + r)^t$ and the general equation for exponential decay is $y = C(1 - r)^t$, where y = the final amount, C = the initial amount, r = the rate of change, and t = the time.

Key Vocabulary

axis of symmetry (p. 472)
completing the square (p. 487)
compound interest (p. 511)
discriminant (p. 496)
exponential function (p. 502)
general equation for exponential decay (p. 511)
general equation for exponential growth (p. 510)
maximum (p. 472)
minimum (p. 472)
parabola (p. 471)
quadratic equation (p. 480)
Quadratic Formula (p. 493)
quadratic function (p. 471)
roots (p. 480)
symmetry (p. 472)
vertex (p. 472)
zeros (p. 480)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. The graph of a quadratic function is a <u>parabola</u>. **true**
2. The solutions of a quadratic equation are called <u>roots</u>. **true**
3. The <u>zeros</u> of a quadratic function can be found by using the equation $x = -\frac{b}{2a}$. **false; axis of symmetry**
4. The <u>vertex</u> is the maximum or minimum point of a parabola. **true**
5. The <u>exponential decay</u> equation is $y = C(1 + r)^t$. **false; exponential growth**
6. An example of a <u>quadratic function</u> is $y = 8^x$. **false; exponential function**
7. <u>Symmetry</u> is a geometric property possessed by parabolas. **true**
8. The graph of a quadratic function has a <u>minimum</u> if the coefficient of the x^2 term is negative. **false; maximum**
9. The expression $b^2 - 4ac$ is called the <u>discriminant</u>. **true**
10. A quadratic equation whose graph has two x-intercepts has <u>no</u> real roots. **false; two**

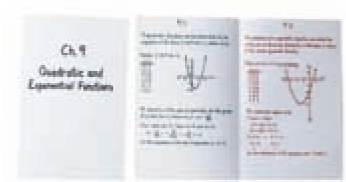
Math Online Vocabulary Review at ca.algebra1.com

CHAPTER 9 Study Guide and Review

FOLDABLES Study Organizer — Dinah Zike's Foldables™

Have students review the chapter to make sure they have included information for every lesson page in their Foldables. Now is a good time to ask if students have any questions about the concepts that they recorded.

Suggest that students keep their Foldables handy while completing the Study Guide and Review pages. Point out that their Foldables can serve as quick review tools for studying for the chapter test.



Formative Assessment

Key Vocabulary The page reference after each word denotes where that term was first introduced. If students have difficulty answering questions 1–10, remind them that they can use these page references to refresh their memories about the vocabulary terms.

Math Online ca.algebra1.com

Vocabulary PuzzleMaker improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work online or from a printed worksheet.

Summative Assessment

CRM Vocabulary Test, p. 54

Chapter 9 Study Guide and Review

Lesson-by-Lesson Review

Intervention If the given examples are not sufficient to review the topics covered by the questions, remind students that the page references tell them where to review that topic in their textbooks.

Two-Day Option Have students complete the Lesson-by-Lesson Review on pp. 518–520. Then you can use ExamView® Assessment Suite to customize another review worksheet that practices all the objectives of this chapter or only the objectives on which your students need more help.

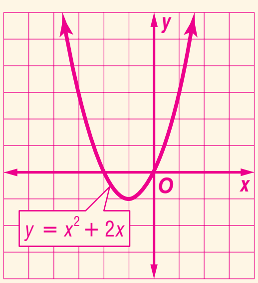
For more information on ExamView® Assessment Suite, see p. 468C.

Differentiated Instruction

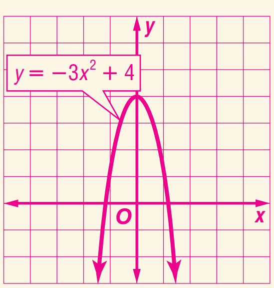
Super DVD: MindJogger Videoquizzes
Use this DVD as an alternative format of review for the test. For more information on this game show format, see p. 468D.

Additional Answers

11. $x = -1$; $(-1, -1)$; min



12. $x = 0$; $(0, 4)$; max



Chapter 9 Study Guide and Review

Lesson-by-Lesson Review

9-1 Graphing Quadratic Functions (pp. 471–477)

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

11. $y = x^2 + 2x$ **12.** $y = -3x^2 + 4$

13. $y = x^2 - 3x - 4$ **14.** $y = 3x^2 + 6x - 17$

15. $y = -2x^2 + 1$ **16.** $y = -x^2 - 3x$

11–16. See margin.

PHYSICAL SCIENCE For Exercises 17–20, use the following information.

A model rocket is launched with a velocity of 64 feet per second. The equation $h = -16t^2 + 64t$ gives the height of the rocket t seconds after it is launched.

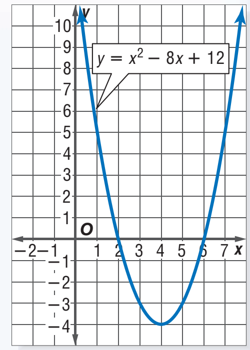
17. Write the equation of the axis of symmetry and find the coordinates of the vertex. **$x = 2$; $(2, 64)$**

18. Graph the function. **See margin.**

19. What is the maximum height that the rocket reaches? **64 ft**

20. How many seconds is the rocket in the air? **4 s**

Example 1 Consider the graph of $y = x^2 - 8x + 12$.



a. Write the equation of the axis of symmetry.

$x = -\frac{b}{2a}$

$x = -\frac{-8}{2(1)}$

$x = 4$

The equation of the axis of symmetry is $x = 4$.

b. Find the coordinates of the vertex.

The x-coordinate for vertex is 4.

$y = x^2 - 8x + 12$ Original equation

$= (4)^2 - 8(4) + 12$ $x = 4$

$= 16 - 32 + 12$ Simplify.

$= -4$ Simplify.

The coordinates of the vertex are $(4, -4)$.

9-2 Solving Quadratic Equations by Graphing (pp. 480–485)

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

21. $x^2 - x - 12 = 0$ **22.** $-x^2 + 6x - 9 = 0$

23. $x^2 + 4x - 3 = 0$ **24.** $2x^2 - 5x + 4 = 0$

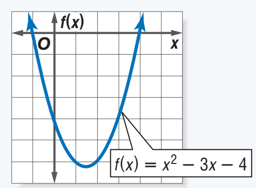
25. $x^2 - 10x = -21$ **26.** $6x^2 - 13x = 15$

25–26. See Ch. 9 Answer Appendix.

27. NUMBER THEORY Use a quadratic equation to find two numbers whose sum is 5 and whose product is −24. **−3 and 8**

Example 2 Solve $x^2 - 3x - 4 = 0$ by graphing.

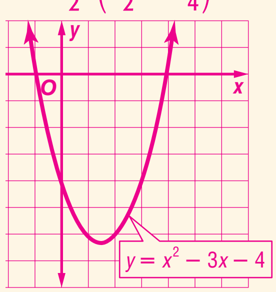
Graph the related function $f(x) = x^2 - 3x - 4$.



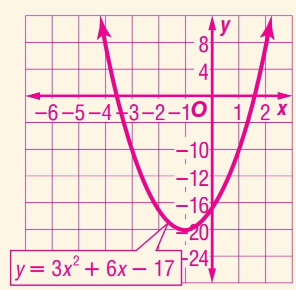
The x-intercepts are −1 and 4. Therefore, the solutions are −1 and 4.

518 Chapter 9 Quadratic and Exponential Functions

13. $x = 1\frac{1}{2}$; $\left(1\frac{1}{2}, -6\frac{1}{4}\right)$; min



14. $x = -1$; $(-1, -20)$; min



Mixed Problem Solving
For mixed problem-solving practice, see page 752.

9-3 Solving Quadratic Equations by Completing the Square (pp. 486–491)

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

28. $a^2 + 6a + 9 = 4$ **−5, −1**
29. $n^2 - 2n + 1 = 25$ **−4, 6**

Solve each equation by completing the square. Round to the nearest tenth if necessary.

30. $-3x^2 + 4 = 0$ **−1.2, 1.2**
31. $x^2 - 16x + 32 = 0$ **2.3, 13.7**
32. $m^2 - 7m = 5$ **−0.7, 7.7**
33. **GEOMETRY** The area of a square can be tripled by increasing the length by 6 centimeters and the width by 3 centimeters. What is the length of the side of the square? **6 cm**

Example 3 Solve $y^2 + 6y + 2 = 0$ by completing the square. Round to the nearest tenth if necessary.

Step 1 Isolate the y^2 and y terms.

$y^2 + 6y + 2 = 0$ Original equation

$y^2 + 6y = -2$ Subtract 2 from each side.

Step 2 Complete the square and solve.

$y^2 + 6y + 9 = -2 + 9$ $\left(\frac{6}{2}\right)^2 = 9$; add 9 to each side.

$(y + 3)^2 = 7$ Factor $y^2 + 6y + 9$.

$y + 3 = \pm\sqrt{7}$ Take the square root of each side.

$y = -3 \pm \sqrt{7}$ Subtract 3 from each side.

The solutions are about −5.6 and −0.4.

9-4 Solving Quadratic Equations by Using the Quadratic Formula (pp. 493–499)

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

34. $x^2 - 8x = 20$ **−2, 10**
35. $r^2 + 10r + 9 = 0$ **−9, −1**
36. $4p^2 + 4p = 15$ **−2.5, 1.5**
37. $2y^2 + 3 = -8y$ **−3.6, −0.4**
38. $2d^2 + 8d + 3 = 3$ **−4, 0**
39. $21a^2 + 5a - 7 = 0$ **−0.7, 0.5**
40. **ENTERTAINMENT** A stunt person attached to a safety harness drops from a height of 210 feet. A function that models the drop is $h = -16t^2 + 210$, where h is the height in feet and t is the time in seconds. About how many seconds does it take to drop from 210 feet to 30 feet? **about 3.4 s**

Example 4 Solve $2x^2 + 7x - 15 = 0$ by using the Quadratic Formula.

For this equation, $a = 2$, $b = 7$, and $c = -15$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$ $a = 2$, $b = 7$, and $c = -15$

$= \frac{-7 \pm \sqrt{169}}{4}$ Simplify.

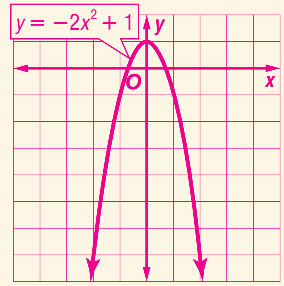
$= \frac{-7 + 13}{4}$ or $\frac{-7 - 13}{4}$ Separate the solutions.

$x = 1.5$ or $x = -5$ Simplify.

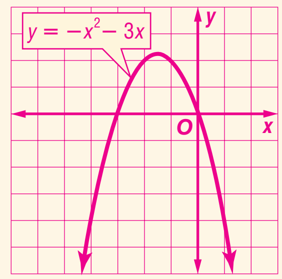
The solutions are −5 and 1.5.

Additional Answers

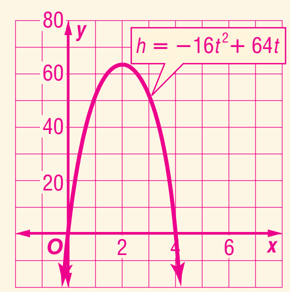
15. $x = 0$; $(0, 1)$; max



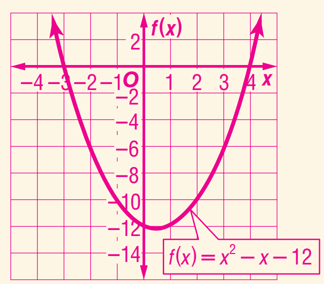
16. $x = -1\frac{1}{2}$; $\left(-1\frac{1}{2}, 2\frac{1}{4}\right)$; max



18.

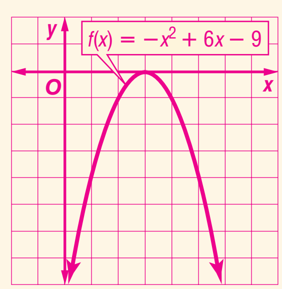


21.



−3, 4

22.



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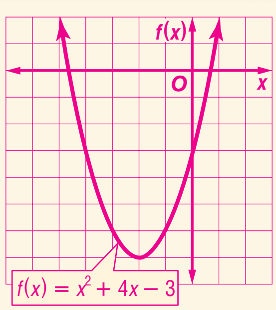
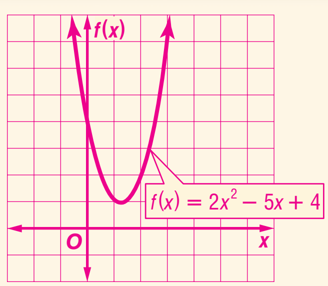
23.



$-5 < x < -4, 0 < x < 1$

24.



∅

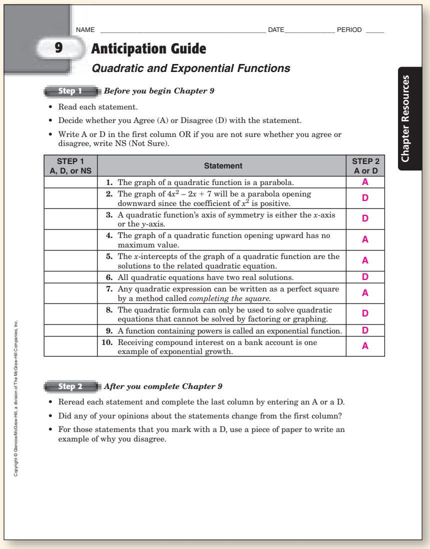
Problem Solving Review

For additional practice in problem solving for Chapter 9, see the Mixed Problem Solving Appendix, p. 752 in the Student Handbook section.

Anticipation Guide

Have students complete the Chapter 9 Anticipation Guide and discuss how their responses have changed now that they have completed Chapter 9.

CRM Anticipation Guide, p. 3


NAME ________________ DATE __________ PERIOD _____

9 **Anticipation Guide**

Quadratic and Exponential Functions

Step 1 *Before you begin Chapter 9*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. The graph of a quadratic function is a parabola.	A
	2. The graph of $4x^2 - 2x + 7$ will be a parabola opening downward since the coefficient of x^2 is positive.	D
	3. A quadratic function's axis of symmetry is either the x-axis or the y-axis.	D
	4. The graph of a quadratic function opening upward has no maximum value.	A
	5. The x-intercepts of the graph of a quadratic function are the solutions to the related quadratic equation.	A
	6. All quadratic equations have two real solutions.	D
	7. Any quadratic expression can be written as a perfect square by a method called *completing the square.*	A
	8. The quadratic formula can only be used to solve quadratic equations that cannot be solved by factoring or graphing.	D
	9. A function containing powers is called an exponential function.	D
	10. Receiving compound interest on a bank account is one example of exponential growth.	A

Step 2 *After you complete Chapter 9*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

Chapter Resources

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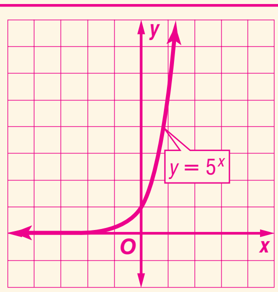
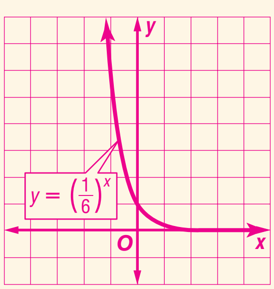
Additional Answers

41.



1; 3.1

42.



1; 0.7

Chapter 9 Study Guide and Review

9-5 Exponential Functions (pp. 502–508)

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value. **41–42. See margin.**

41. $y = 5^x$; $5^{0.7}$ **42.** $y = \left(\frac{1}{6}\right)^x$; $\left(\frac{1}{6}\right)^{0.2}$

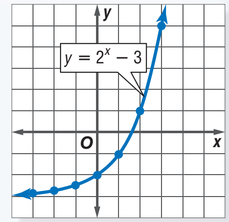
Graph each function. State the y-intercept. **43–44. See margin.**

43. $y = 3^x + 6$ **44.** $y = 3^{x+2}$

45. BIOLOGY The population of bacteria in a Petri dish increases according to the model $p = 550 \cdot 2.7^{0.008t}$, where t is the number of hours and $t = 0$ corresponds to 1:00 P.M. Use this model to estimate the number of bacteria in the dish at 5 P.M. **about 568**

Example 5 Graph $y = 2^x - 3$. State the y-intercept.

x	$2^x - 3$	y
−3	$2^{-3} - 3$	−2.875
−1	$2^{-1} - 3$	−2.5
0	$2^0 - 3$	−2
1	$2^1 - 3$	−1
2	$2^2 - 3$	1
3	$2^3 - 3$	5



Graph the ordered pairs and connect the points with a smooth curve. The y-intercept is −2.

9-6 Growth and Decay (pp. 510–514)

Determine the final amount for each investment.

	Principal	Annual Interest Rate	Time (yr)	Type of Compounding
46.	\$2000	3%	8	quarterly
47.	\$5500	2.25%	15	monthly
48.	\$15,000	2.5%	25	monthly
49.	\$500	1.75%	40	daily

RESTAURANTS For Exercises 50 and 51, use the following information.

The total restaurant sales in the United States increased at an annual rate of about 5.2% between 1996 and 2004. In 1996, the total sales were \$310 billion.

50. Write an equation for the average sales per year for t years after 1996.

51. Predict the total restaurant sales in 2008. **about \$570 billion**

Example 6 Find the final amount of an investment if \$1500 is invested at an interest rate of 2.5% compounded quarterly for 10 years.

$A = p\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest equation

$= 1500\left(1 + \frac{0.025}{4}\right)^{4(10)}$ $P = 1500$, $r = 2.5\%$ or 0.025, $n = 4$, and $t = 10$

≈ 1924.54 Simplify.

The final amount in the account is about \$1924.54.

46. \$2540.22
47. \$7705.48
48. \$28,005.48
49. \$1006.86
50. $y = 310(1.052)^t$

Problem Solving Review

For additional practice in problem solving for Chapter 9, see the Mixed Problem Solving Appendix, p. 752 in the Student Handbook section.

Anticipation Guide

Have students complete the Chapter 9 Anticipation Guide and discuss how their responses have changed now that they have completed Chapter 9.

CRM Anticipation Guide, p. 3

9 Practice Test

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. **1–4. See margin.**

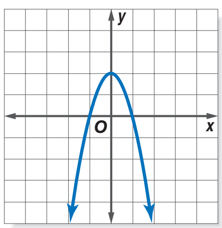
1. $y = x^2 - 4x + 13$
2. $y = -3x^2 - 6x + 4$
3. $y = 2x^2 + 3$
4. $y = -1(x - 2)^2 + 1$

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

5. $x^2 - 2x + 2 = 0$ **$\varnothing$**
6. $x^2 + 6x = -7$ **$-5 < x < -4$; $-2 < x < -1$**
7. $x^2 + 24x + 144 = 0$ **-12**
8. $2x^2 - 8x = 42$ **$-3, 7$**

5–8. See Ch. 9 Answer Appendix for graphs.

9. **MULTIPLE CHOICE** Which function is graphed below? **D**



A $y = 2x^2 - 2$

B $y = 2x^2 + 2$

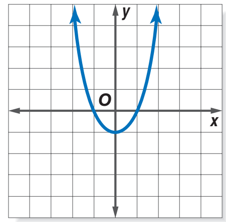
C $y = -2x^2 - 2$

D $y = -2x^2 + 2$

Solve each equation. Round to the nearest tenth if necessary. **10–17. See margin.**

10. $x^2 + 7x + 6 = 0$
11. $2x^2 - 5x - 12 = 0$
12. $6n^2 + 7n = 20$
13. $3k^2 + 2k = 5$
14. $y^2 - \frac{3}{5}y + \frac{2}{25} = 0$
15. $-3x^2 + 5 = 14x$
16. $z^2 - 13z = 32$
17. $7m^2 = m + 5$

18. **MULTIPLE CHOICE** Which equation best represents the parabola graphed below if it is shifted 3 units to the right? **H**



F $y = x^2 - 1$

G $y = x^2 + 2$

H $y = x^2 - 6x + 8$

J $y = x^2 + 6x + 8$

19–22. See Ch. 9 Answer Appendix for graphs.

Graph each function. State the y-intercept.

19. $y = \left(\frac{1}{2}\right)^x$ **1**
20. $y = 4 \cdot 2^x$ **4**
21. $y = 0.5(4^x)$ **0.5**
22. $y = 5^x - 4$ **-3**

23. Graph $y = \left(\frac{1}{3}\right)^x - 3$ and state the y-intercept. Then use the graph to determine the approximate value of $y = \left(\frac{1}{3}\right)^{3.5} - 3$. Use a calculator to confirm the value.
-2, -3.0; See Ch. 9 Answer Appendix for graph.

24. **CARS** Ley needs to replace her car. If she leases a car, she will pay \$410 a month for 2 years and then has the option to buy the car for \$14,458. The current price of the car is \$17,369. If the car depreciates at 16% per year, how will the depreciated price compare with the buy-out price of the lease?
\$2002 less than the buyout price

25. **FINANCE** Find the total amount of the investment shown in the table if interest is compounded quarterly. **about \$2721.03**

Principal	\$1500
Length of Investment	10 yr
Annual Interest Rate	6%

9 Practice Test

Summative Assessment

Chapter 9 Resource Masters

Leveled Chapter 9 Tests

Form	Type	Level	Pages
1	MC	BL	55–56
2A	MC	OL	57–58
2B	MC	OL	59–60
2C	FR	OL	61–62
2D	FR	OL	63–64
3	FR	AL	65–66

MC = multiple-choice questions

FR = free-response questions

BL = below grade level

OL = on grade level

AL = above grade level

- Vocabulary Test, p. 54
- Extended-Response Test, p. 67

ExamView Assessment Suite Customize and create multiple versions of your chapter tests and their answer keys. All of the questions from the leveled chapter tests in the *Chapter 9 Resource Masters* are also available on ExamView Assessment Suite with the California Standard that each item assesses.

Additional Answers

1–4. See Chapter 9 Answer Appendix for graphs.

1. $x = 2$; $(2, 9)$; min
2. $x = -1$; $(-1, 7)$; max
3. $x = 0$; $(0, 3)$; min
4. $x = 2$; $(2, 1)$; max

Data-Driven Decision Making	Exercises	Lesson	Standard	Resources for Review
Diagnostic Teaching Based on the results of the Chapter 9 Practice Test, use the following to review concepts that students continue to find challenging.	10–17	9-4	20.0	CRM Study Guide and Intervention pp. 27-28, 34–35, 42–43; Math Online • Extra Examples • Personal Tutor • Concepts in Motion
	19–23	9-5	Preparation for Algebra II 12.0	
	24–25	9-6	Preparation for Algebra II 12.0	

Chapter 9 California Standards Practice

Formative Assessment

You can use these two pages to benchmark student progress. The California Standards are listed with each question.

Chapter 9 Resource Masters
- Standardized Test Practice, pp. 68–70

ExamView Assessment Suite

Create practice worksheets or tests that align to the California Standards, as well as TIMSS and NAEP tests.

Chapter 9 California Standards Practice

Cumulative, Chapters 1–9

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1 Trevor is solving this equation by completing the square.

$$rx^2 + sx + t = 0 \text{ (where } r \geq 0)$$

Step 1: $rx^2 + sx = -t$

Step 2: $x^2 + \frac{s}{r}x = -\frac{t}{r}$

Step 3: ?

Which should be Step 3 in the solution? **D**

A $x^2 = -\frac{s}{r} - \frac{s}{r}x$

B $x^2 + \frac{s}{r}x + \frac{s}{2r} = -\frac{t}{r} + \frac{s}{2r}$

C $x + \frac{s}{r} = -\frac{t}{rx}$

D $x^2 + \frac{s}{r}x + \left(\frac{s}{2r}\right)^2 = -\frac{t}{r} + \left(\frac{s}{2r}\right)^2$

2 How many times does the graph of $y = 3x^2 - 3x + 4$ intersect the x-axis? **F**

F none H two
G one J three

3 An object that is projected straight downward with initial velocity v feet per second travels a distance $s = vt + 16t^2$, where t = time in seconds. Rafael is in a hot air balloon 204 feet above the ground and throws a penny straight down with an initial velocity of 20 feet per second. In how many seconds will the penny reach the ground? **A**

A 3 C 5
B 4 D 6

TEST-TAKING TIP

Question 3 Always write down your calculations on scrap paper or in the test booklet, even if you think you can do the calculations in your head. Writing down your calculations will help you avoid making simple mistakes.

4 Which is a factor of $x^2 + x - 20$? **G**

F $x - 5$
G $x + 5$
H $x - 6$
J $x + 6$

5 Find the product of $(3m - 4)(m + 5)$. **A**

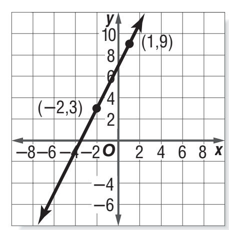
A $3m^2 + 11m - 20$
B $3m^2 + 11m + 20$
C $3m^2 + 19m - 20$
D $3m^2 + 19m + 20$

6 What is the solution to this system of equations? **G**

$$4x + 7y = 9$$
$$-x - 5y = -12$$

F $(-3, -3)$
G $(-3, 3)$
H $(3, -3)$
J $(3, 3)$

7 What is the slope of the line graphed below? **C**



A −6
B −2
C 2
D 6

Math Online California Standards Practice at ca.algebra1.com

522 Chapter 9 Quadratic and Exponential Functions

More California Standards Practice
For practice by standard, see pages CA1–CA43.

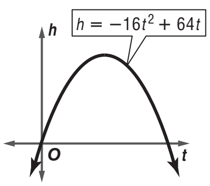
8 When is this statement true? **H**

The multiplicative inverse of a number is less than the original number.

F This statement is never true.
G This statement is always true.
H This statement is true for numbers greater than 1.
J This statement is true for numbers less than −1.

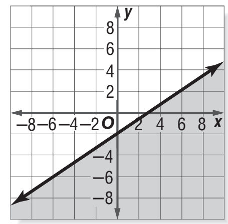
9 The length of a rectangle is three more than twice the width. If the perimeter of the rectangle is 15, what is the length of the rectangle? **6**

10 Jennie hit a baseball straight up in the air. The height h, in feet, of the ball above the ground is modeled by the equation $h = -16t^2 + 64t$. How long is the ball above ground? **C**



A 1 second
B 2 seconds
C 4 seconds
D 16 seconds

11 Which inequality is shown on the graph below? **F**



F $y \leq \frac{2}{3}x - 2$

G $y < \frac{2}{3}x - 2$

H $y \geq \frac{2}{3}x - 2$

J $y > \frac{2}{3}x - 2$

Pre-AP/Anchor Problem

Record your answers on a sheet of paper. Show your work.

12 Annika Sorenstam hit a golf ball and its path through the air is modeled by the equation $h = -16t^2 + 80t + 1$, where h is the height in feet of the golf ball and t is the time in seconds.

a. Approximately how long was the ball in the air? **5.01 seconds**

b. How high was the ball at its highest point? **101.0 feet**

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	9–3	9–2	9–4	8–3	7–6	5–4	4–1	1–4	5–2	9–4	6–6	9–2
For Help with Standard...	9.0	22.0	23.0	11.0	10.0	8.0	7AF3.3	25.1	9.0	23.0	6.0	23.0

Chapter 9 California Standards Practice

TEST-TAKING TIP

Question 7 One advantage of this multiple-choice question is that a correct answer is among the choices. When this is the case, it is sometimes possible to work backward by taking the answers and testing each of them in the original problem.

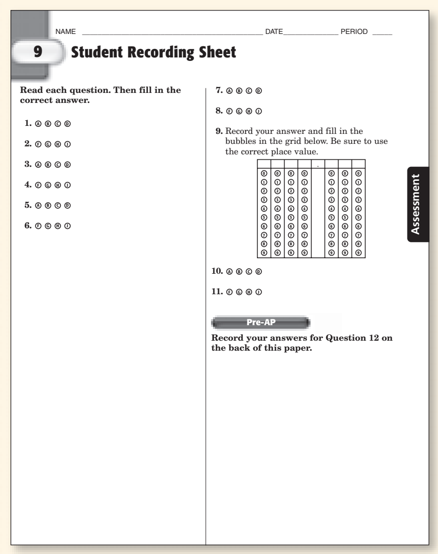
Item Analysis

Question 9 is a griddable question. In griddable questions, students arrive at an answer and then record it in a special grid, coloring in the appropriate bubble under each digit of the answer.

Answer Sheet Practice

Have students simulate taking a standardized test by recording their answers on practice recording sheets.

CRM Student Recording Sheet, p. 49


NAME ______ DATE ______ PERIOD ______

9 **Student Recording Sheet**

Read each question. Then fill in the correct answer.

1. ○○○○
2. ○○○○
3. ○○○○
4. ○○○○
5. ○○○○
6. ○○○○
7. ○○○○
8. ○○○○
9. Record your answer and fill in the bubbles in the grid below. Be sure to use the correct place value.
10. ○○○○
11. ○○○○

Pre-AP

Record your answers for Question 12 on the back of this paper.

Assessment

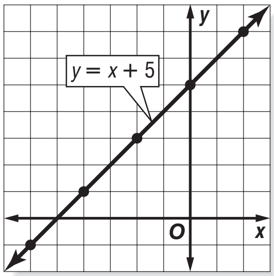
Homework Option

Get Ready for Chapter 10 Assign students the exercises on p. 519 as homework to assess whether they possess the prerequisite skills needed for the next chapter.

Page 469, Get Ready for Chapter 9

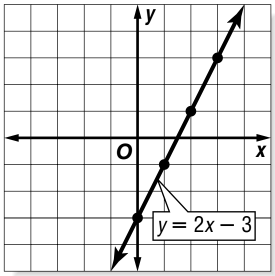
1.

x	y
−6	−1
−4	1
−2	3
0	5
2	7



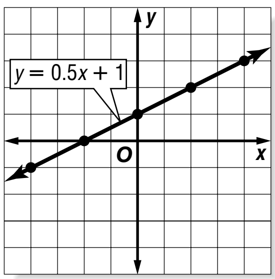
2.

x	y
0	−3
1	−1
2	1
3	3



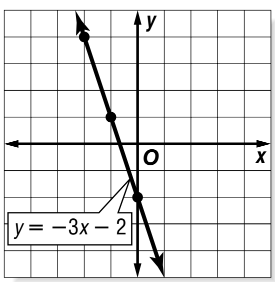
3.

x	y
−4	−1
−2	0
0	1
2	2
4	3



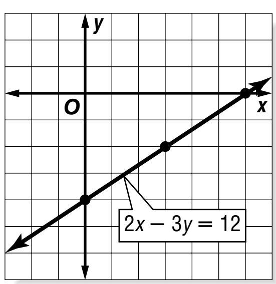
4.

x	y
−2	4
−1	1
0	−2



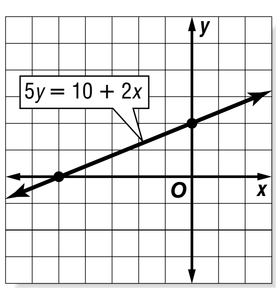
5.

x	y
0	−4
3	−2
6	0



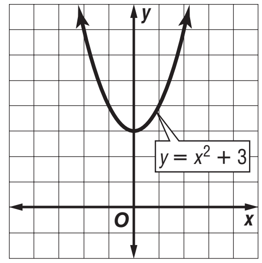
6.

x	y
−5	0
0	2



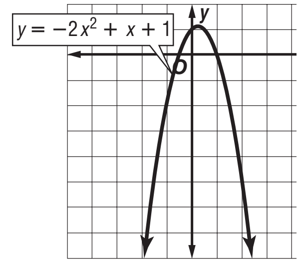
Page 472, Lesson 9-1 (Check Your Progress)

1.



D: $\{x \mid x$ is a real number$\}$;
R: $\{y \mid y \geq 3\}$

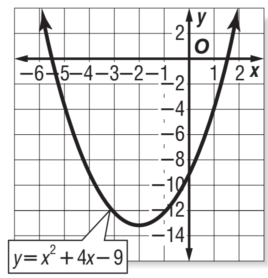
2.



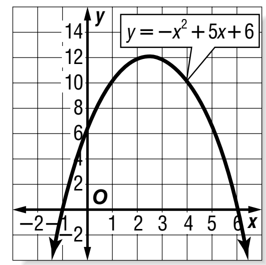
D: $\{x \mid x$ is a real number$\}$;
R: $\left\{y \mid y \geq 1\frac{1}{8}\right\}$

Pages 475-476, Lesson 9-1

5.



6.



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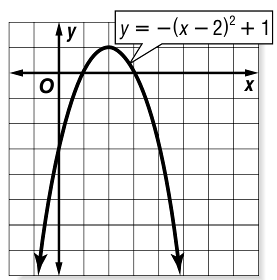
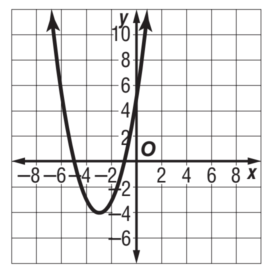
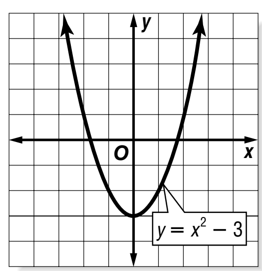


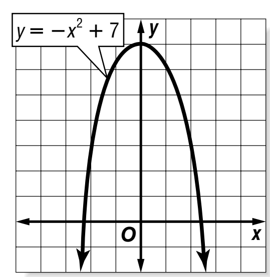
8.



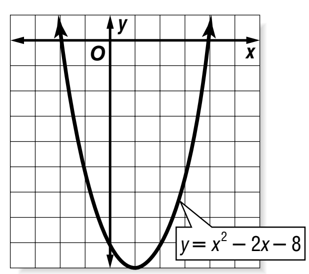
10.



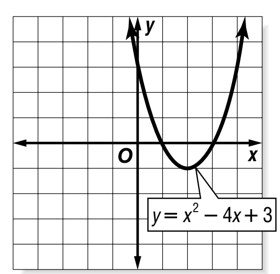
11.



12.



13.



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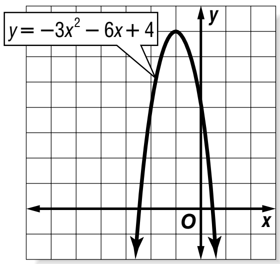

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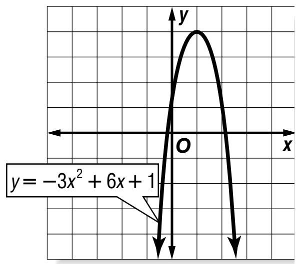

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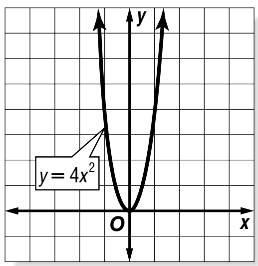

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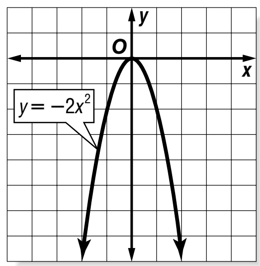
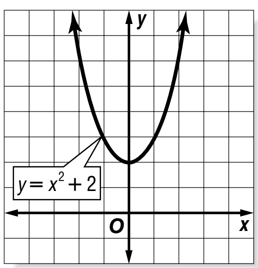
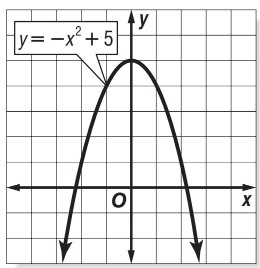


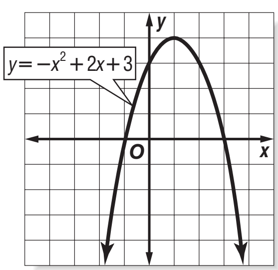
18.



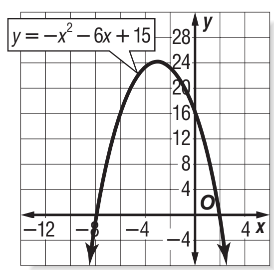
19.



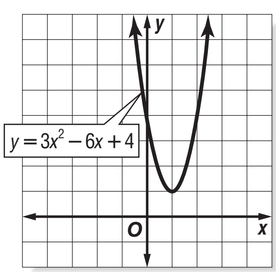
20.



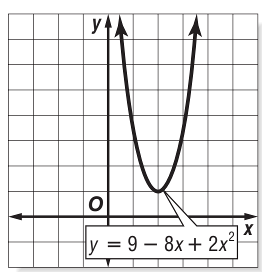
21.



22.

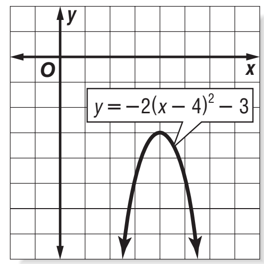


23.

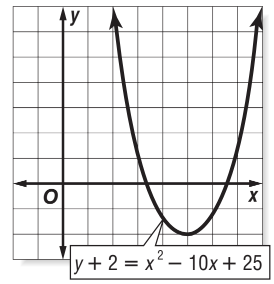


28. $A = x(20 - x)$ or $A = -x^2 + 20x$; A reasonable domain is $\{x \mid 0 < x < 20\}$ because if x is less than or equal to 0 or greater than or equal to 20, the area has a value of zero or it has a negative value. A reasonable range is $\{y \mid 0 < y < 100\}$, because these are the values when x is between 0 and 20.

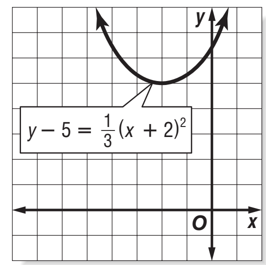
30.



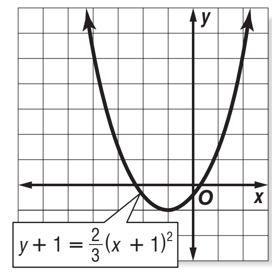
31.



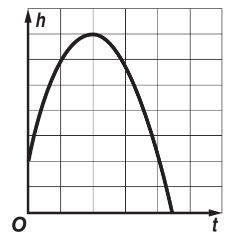
32.



33.

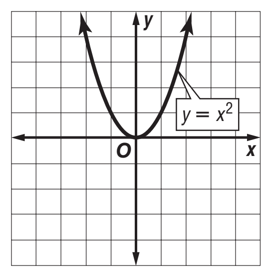


42. Sample answer:



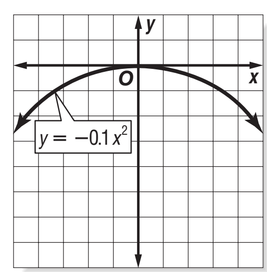
The vertex represents the height of a ball after it is tossed up in the air. Reasonable domain and range values are real numbers greater than or equal to 0.

43.



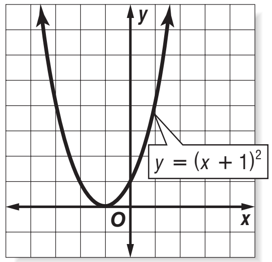
Page 479, Extend 9-1

5. The graph will have a vertex at the origin, open downward, and be wider than $y = -x^2$.

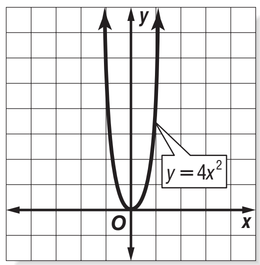


Chapter 9 Answer Appendix

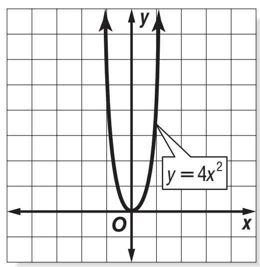
6. The graph will open upward and have the same shape as $y = x^2$, but the vertex will be at $(-1, 0)$.



7. The graph will open upward, have a vertex at the origin, and be narrower than $y = x^2$.

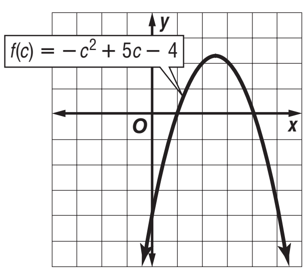


8. The graph will open upward and have the same shape as $y = x^2$, but its vertex will be at $(0, -6)$.

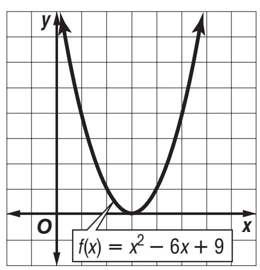


Page 481, Lesson 9-2 (Check Your Progress)

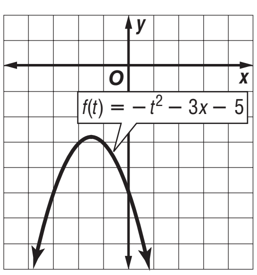
1.



2.

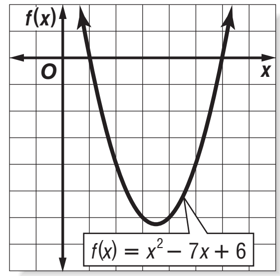


3.



Pages 483–485, Lesson 9-2

1.



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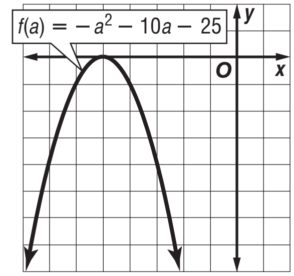

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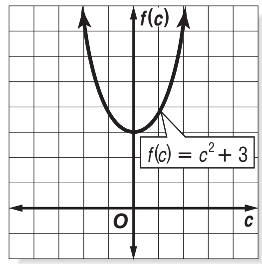

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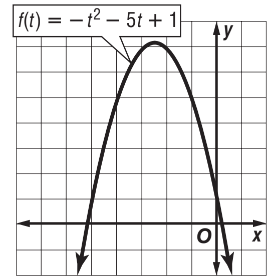

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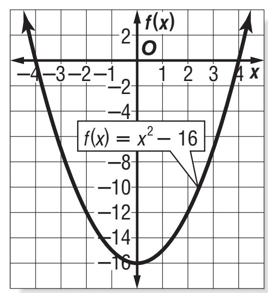

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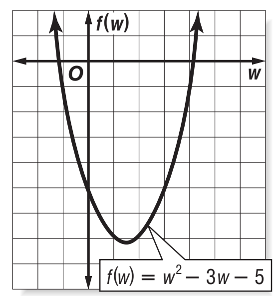

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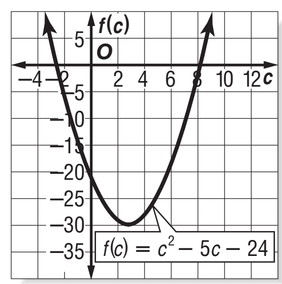

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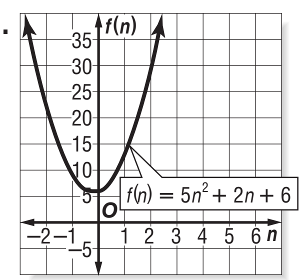

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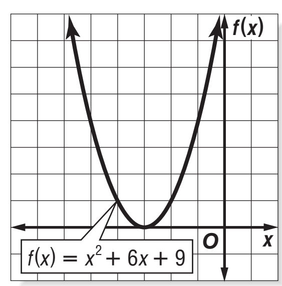

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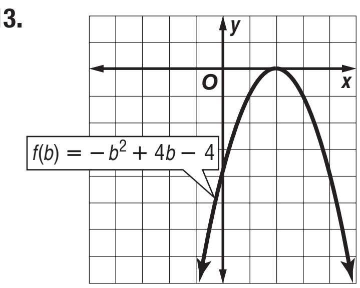

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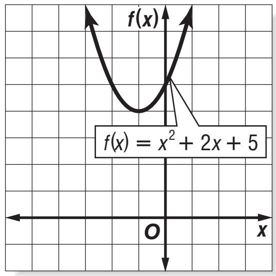

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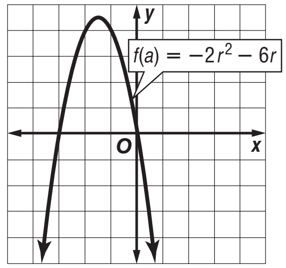

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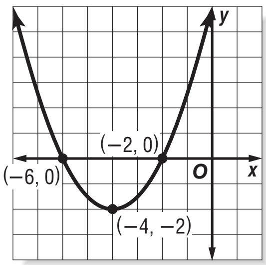
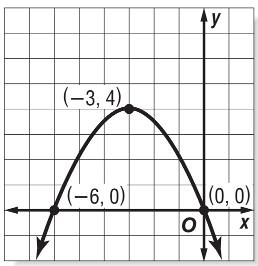


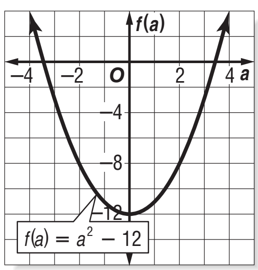
The graph is wider than the graph of $y = x^2$. Also, the vertex is 4 units left and 2 units down from the vertex of the graph of $y = x^2$.

17.



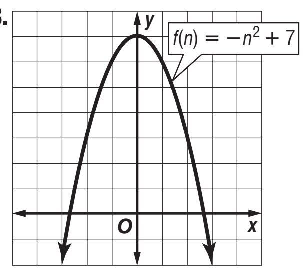
The graph is wider than the graph of $y = x^2$ and opens downward rather than upward. Also, the vertex is 3 units left and 4 units up from the vertex of the graph of $y = x^2$.

22.



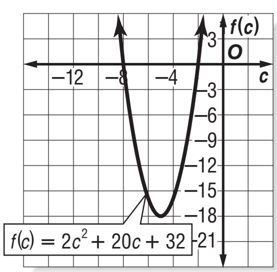
$-4 < a < -3, 3 < a < 4$

23.



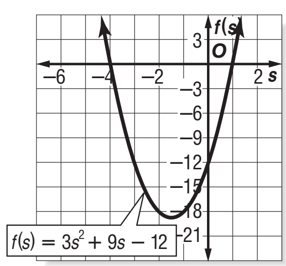
$-3 < n < -2, 2 < n < 3$

24.



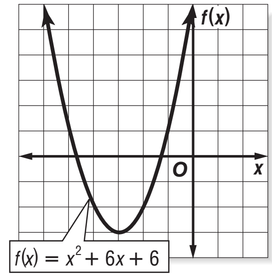
$-8, -2$

25.



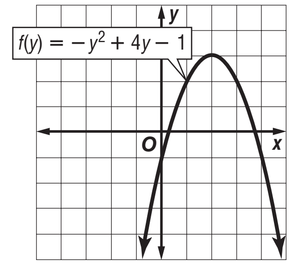
$-4, 1$

26.



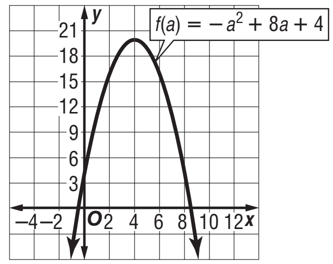
$-5 < x < -4, -2 < x < -1$

27.



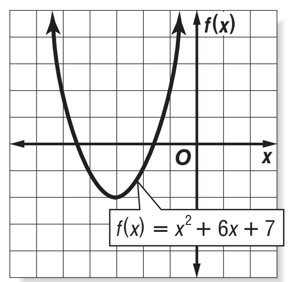
$0 < y < 1, 3 < y < 4$

28.



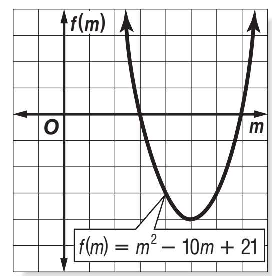
$-1 < a < 0, 8 < a < 9$

29.



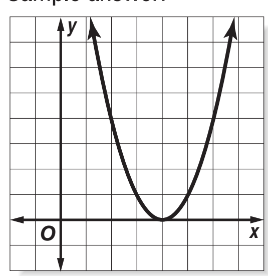
$-5 < x < -4, -2 < x < -1$

30.



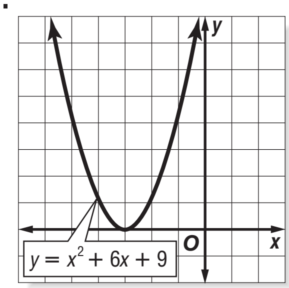
3, 7

41. Sample answer:



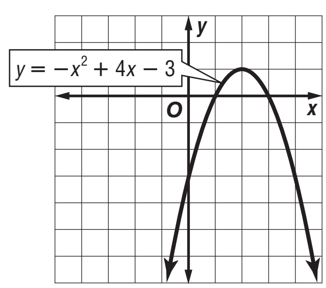
The only solution to the equation with the graph shown is 4.

47.

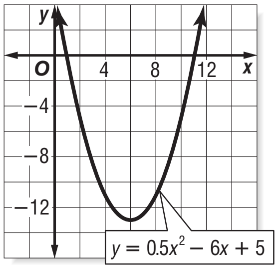


$x = -3$; $(-3, 0)$; min

48.

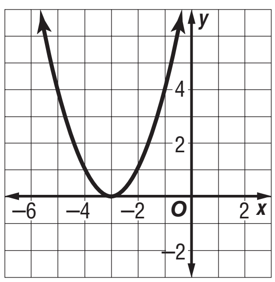


$x = 2$; $(2, 1)$; max

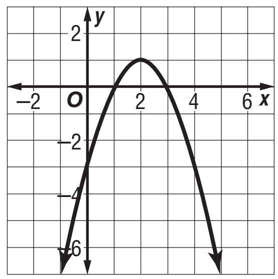
49.


$x = 6$; $(6, -13)$; min

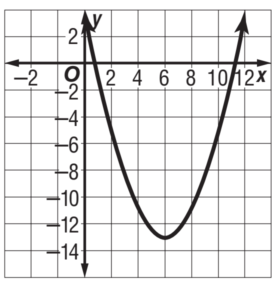
Page 485, Lesson 9-2

47.


$x = -3$; $(-3, 0)$; min

48.


$x = 2$; $(2, 1)$; max

49.


$x = 6$; $(6, -13)$; min

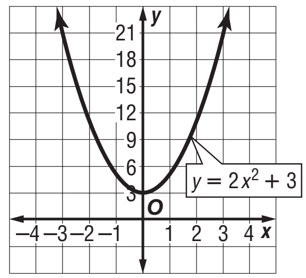
Page 491, Lesson 9-3

47. Sample answer: Al-Khwarizmi used squares to geometrically represent quadratic equations. He represented x^2 by a square whose sides were each x units long. To this square, he added 4 rectangles with length x units long and width $\frac{8}{4}$ or 2 units long. This area represents 35. To make this a square, four 4×4 squares must be added. To solve $x^2 + 8x = 35$ by completing the square, use the following steps.

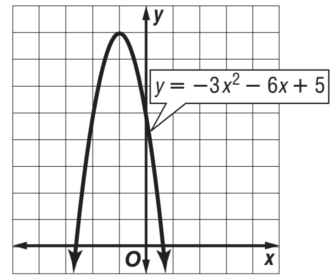
$x^2 + 8x = 35$	Original equation.
$x^2 + 8x + 16 = 35 + 16$	Since $\left(\frac{8}{2}\right)^2 = 16$, add 16 to each side.
$(x + 4)^2 = 51$	Factor $x^2 + 8x + 16$.
$x + 4 = \pm\sqrt{51}$	Take the square root of each side.
$x + 4 - 4 = \pm\sqrt{51} - 4$	Subtract 4 from each side.
$x = -4 \pm \sqrt{51}$	Simplify.
$x = -4 - \sqrt{51}$ or $x = -4 + \sqrt{51}$	Simplify.
$x \approx -11.14$	$x \approx 3.14$

The solutions are -11.14 and 3.14.

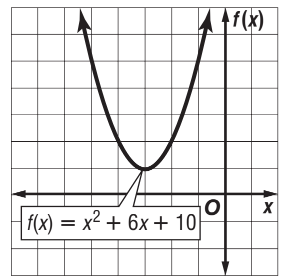
Page 492, Mid-Chapter Quiz

2.


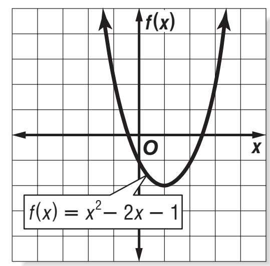
$x = 0$; $(0, 3)$; minimum

3.


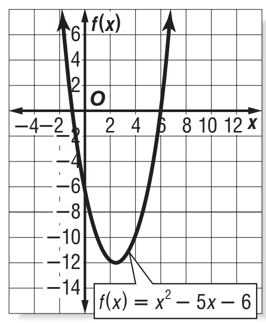
$x = -1$; $(-1, 8)$; maximum

5.


$\varnothing$

6.


$-1 < x < 0$, $2 < x < 3$

7.


-1, 6

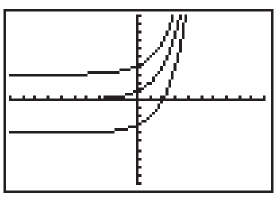
Page 498, Lesson 9-4

43. The function can be factored to $f(x) = (x - 4)^2$, so there is one real root at (4, 0). Using the discriminant to determine the number of roots involves more computation and potential for error.

44. Sample answer: (1) Factor $x^2 - 2x - 15$ as $(x + 3)(x - 5)$. Then according to the Zero Product Property, either $x + 3 = 0$ or $x - 5 = 0$. Solving these equations, $x = -3$ or $x = 5$. (2) Rewrite the equation as $x^2 - 2x = 15$. Then add 1 to each side of the equation to complete the square on the left side. Then $(x - 1)^2 = 16$. Taking the square root of each side, $x - 1 = \pm 4$. Therefore, $x = 1 \pm 4$ and $x = -3$ or $x = 5$. (3) Use the Quadratic Formula. Therefore, $x = \frac{2 \pm \sqrt{2^2 - 4(1)(-15)}}{2(1)}$ or $x = \frac{2 \pm \sqrt{64}}{2}$. Simplifying the expression, $x = -3$ or $x = 5$. See students' preferences.

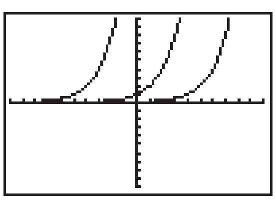
Page 504, Lesson 9-5 (Graphing Calculator Lab)

1. The graphs are the same shape. The graph of $y = 2^x + 3$ is the graph of $y = 2^x$ translated 3 units up. The graph of $y = 2^x - 4$ is the graph of $y = 2^x$ translated 4 units down.



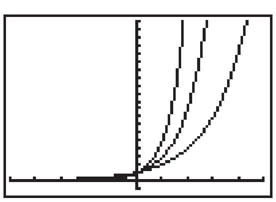
[−10, 10] scl: 1 by [−10, 10] scl: 1

2. The graphs are the same shape. The graph of $y = 2^{x+5}$ is the graph of $y = 2^x$ translated 5 units to the left. The graph of $y = 2^{x-4}$ is the graph of $y = 2^x$ translated 4 units to the right.



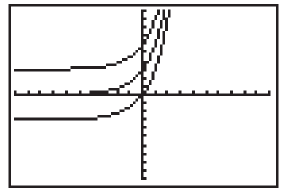
[−10, 10] scl: 1 by [−10, 10] scl: 1

3. All of the graphs cross the y-axis at 1. The graph of $y = 3^x$ is steeper than the graph of $y = 2^x$, and the graph of $y = 5^x$ is steeper yet.



[−5, 5] scl: 1 by [−1, 20] scl: 1

4. The graphs are the same shape. The graph of $y = 3(2^x - 1)$ is the graph of $y = 3(2^x)$ translated 3 units down. The graph of $y = 3(2^x + 1)$ is the graph of $y = 3(2^x)$ translated 3 units up.



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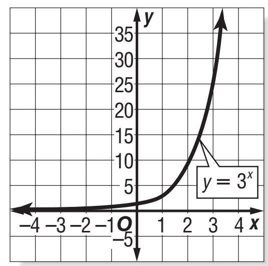
Pages 505–507, Lesson 9-5

1.



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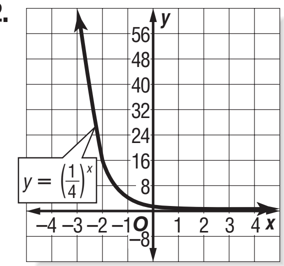

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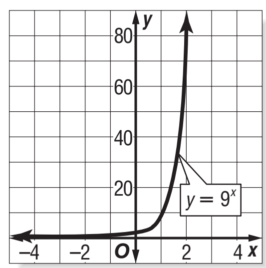

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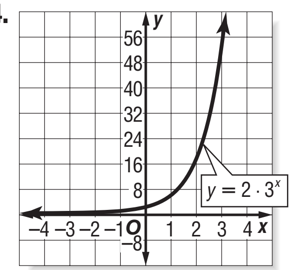

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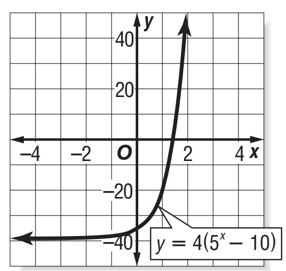

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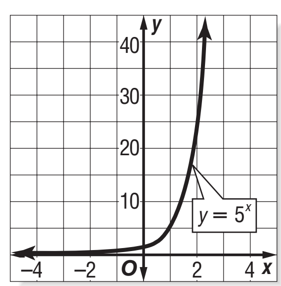

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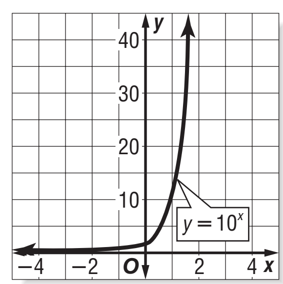

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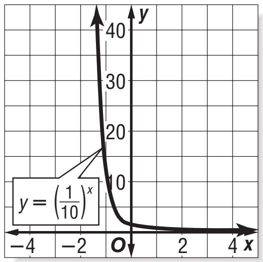

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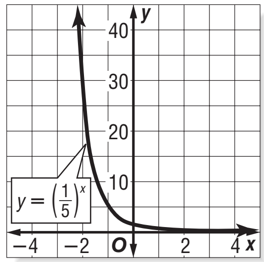

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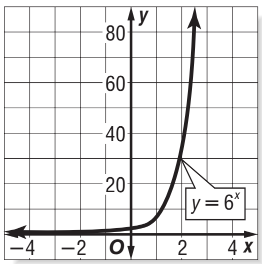

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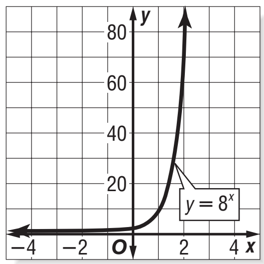

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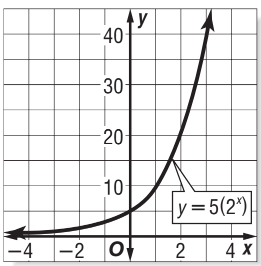

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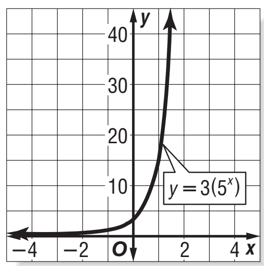

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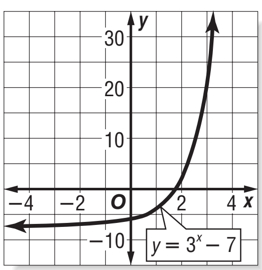

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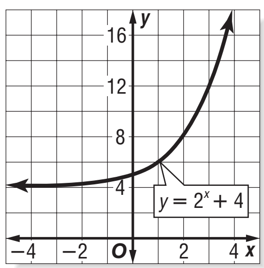

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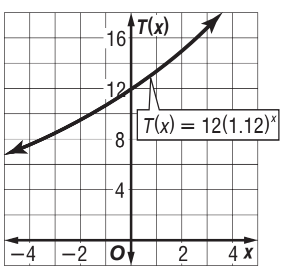

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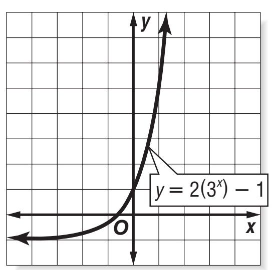

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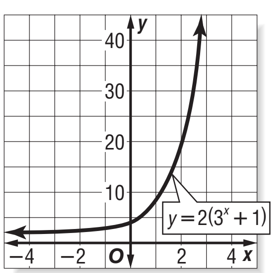

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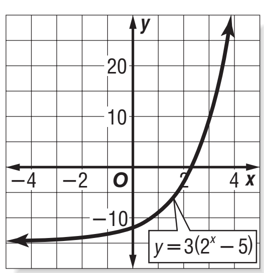
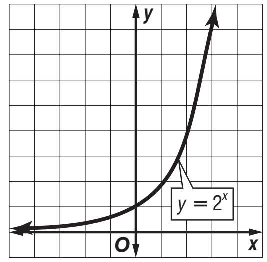
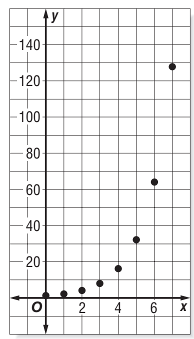


42. Sample answer: The number of teams competing in a basketball tournament can be represented by $y = 2^x$, where the number of teams competing is y and the number of rounds is x.



The y-intercept of the graph is 1. The graph increases quickly for $x > 0$.

Page 509, Explore 9-6

3.


9.

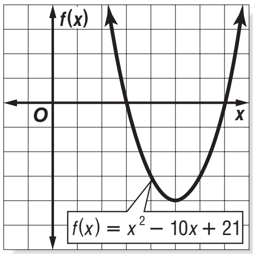
Number of Cuts	Area of Sheet
0	1
1	0.5
2	0.25
3	0.125
4	0.0625
5	0.03125
6	0.015625
7	0.0078125

Page 514, Lesson 9-6

18. Sample answer: If the number of blogs is growing by the same percent each month, an exponential equation can be used to model blogs and predict future blogs. The equation states that the new value equals the amount in November 2003 or 1.1 times the sum of 1 plus 13.7% raised to the power that is equal to the number of months since November 2003. According to the equation, there will be about 14,712 million blogs in January 2010.

Page 518, Study Guide and Review

25.



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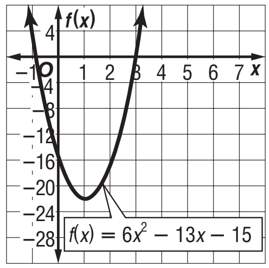
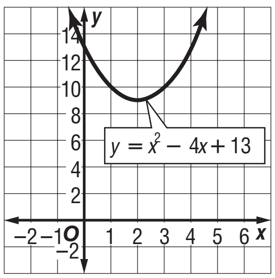


Page 521, Practice Test

1.



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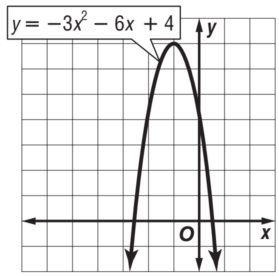

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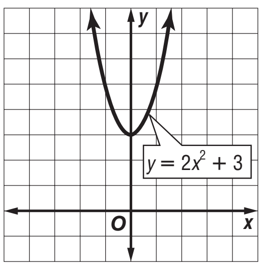

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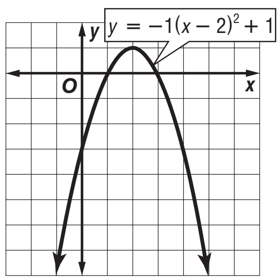

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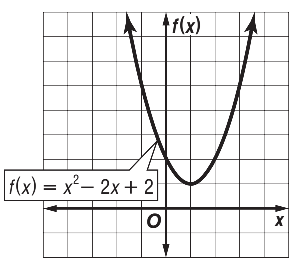

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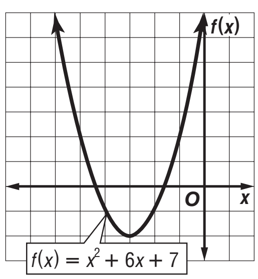

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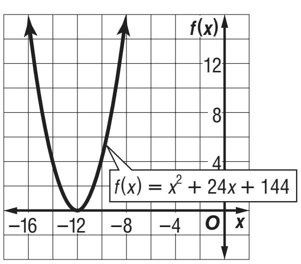
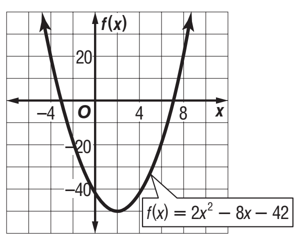


8.



10. $-6, -1$

11. $-1\frac{1}{2}, 4$

12. $-2\frac{1}{2}, 1\frac{1}{3}$

13. $-1\frac{2}{3}, 1$

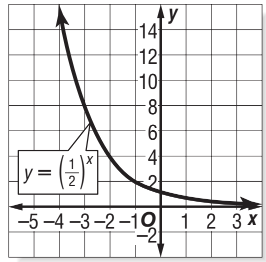
14. $\frac{1}{5}, \frac{2}{5}$

15. $-5, \frac{1}{3}$

16. $-2.1, 15.1$

17. $-0.8, 0.9$

19.



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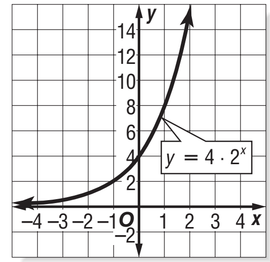

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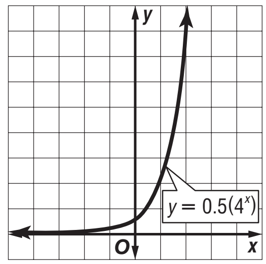

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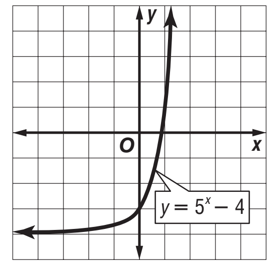

23.